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100 Mock Quantitative Ability Questions for MBA

The Quantitative Ability section is one of the most crucial sections in any management entrance exam. It is popularly known as QA and you can find it in exams such as CAT, XAT, IIFT, SNAP, NMAT, CMAT, and MAT.
The Quantitative Ability section covers topics like: Averages, Geometric Progression, Geometry, HCF & LCM, Inequalities, In-equations Quadratic and linear equations, Logarithms, Mensuration, Number system, Partnership (Accounts), Percentages, Profit & Loss, Ratios and Proportion, Surds and Indices, Time-Speed-Distance, Trigonometry, and Work and time

MBA Rendezvous provides you with 100 questions in QA with their solution. The answers unlock after you attempt the questions.

Question:

1. If $4^{x-5+y} = 2^{x+y} \times 2^{y} \times 2^{x-4} - 63$, find the sum of x and y.

(a) 2 (b) 3 (c) 4 (d) 5

Answer:

1. (d)

2. Five apples and four oranges cost as much as three apples and seven oranges. Find out the ratio of the cost of one apple to the cost of one orange.

(a)3:1 (b) 1:3 (c) 3:2 (d) 2:3

3. Given: $2x \geq 2^{x-1} + 2^{x-2} + \dots\dots$ upto2^{0} If x is an integer then find the value(s) of x.

(a) X=1 only (b) x=1, 2 only (c) x < 3 (d) x > 3

4. If 4x-8+a = bx-1 has an integer solution (a,b) then the values of a and b could be

(a) (4,1) (b) (2,4) (c) (4,2) (d) (6,2)

5. If x+y+z = 25, (x,y,z >0), then the maximum value of (x+y)(y+3)(z) will be

(a) 1000 (b) 3125/27 (c) 3125 (d) ∞

6. Let x be a positive integer such that x+7 is divisible by 5. Find the smallest positive integer n where n>2 such that $x + n^2$ is divisible by 5.

(a) 3 (b) 4 (c) 5 (d) None

7. Find the even factors of 504.

(a) 12 (b) 15 (c) 18 (d) 24

8. If $f(x) = 2x^3 - x + 2k$ and f (1) & f (2) are of opposite signs, then which of the following is true?

(a) $-7<k<1$ (b) $-5<k<1/2$ (c) $-7<k<-1/2$ (d) $-5<k<3/2$

Answer the questions 9 & 10 based on the following information:

H(a, b, c) = Greatest common divisor of a, b, c

L(a, b, c) = Least common multiple of a, b, c

A(a, b, c) = Average of a, b, c

Min(a, b, c) = Smallest value among a, b, c

Max(a, b, c) = Largest among a, b, c

9. If a, b, c are distinct positive real numbers then which of the following is true?

(a) H(a, b, c) × L(a, b, c) = abc *(b) H(a, b, c)>L(a, b, c)*

(c) H(a, b, c)>Min(a, b, c) *(d) H(a, b, c) < A(a, b, c)< L(a, b, c)*

10. If Max(a, b, c) = Min(a, b, c), then

(a) A(a, b, c) = H(a, b, c) *(b) A(a, b, c) =L(a, b, c)*

(c) A(a, b, c) = Min(a, b, c) *(d) All of these*

11. Each root of the equation $ax^3 – 7x^2 + cx + 231 = 0$ is an integer. One of the roots is −1/2 times the sum of the other two roots. What is the sum of all the possible values of a?

(a) 17 (b) – 7 (c) – 17 (d) None of these

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12. $M=\sqrt{3-\sqrt{5}+\sqrt{9-4\sqrt{5}}}$ and $N=\sqrt{\sqrt{7}-1-\sqrt{11-4\sqrt{7}}}$. What is the value of $\frac{M-N}{M+N}$?

(a) 0 (b) 1 (c) – 1 (d) None of these

13. P + 1/Q = Q + 1/R = 1where P, Q and R are real numbers. What is the value of PQR + R + 1/P?

(a) –2 (b) – 1 (c) 0 (d) None of these

14. N = 70! × 69! × 68! × ..... 3! × 2! × 1! Which of the following represents the 147th digit from the right end of N?

(a) 2 (b) 0 (c) 5 (d) 7

15. A 3-digit natural number 'abc', where a, b and c are distinct digits, when increased by 33.33% becomes 'cab'. When 'cab' is increased by 33.33% it becomes 'bca'. How many such numbers are there?

(a) 0 (b) 1 (c) 2 (d) 5

16. If a and b are real numbers such that $a_a b$ and $a \neq b$, then what is the value of $a^b - b$?

(a) – 1 (b) 0 (c) 1 (d) 2

17. A function f(x) is defined for all real values of x as x 1 f(x) = (x − 1)/(x + 1) . If $y_1 = f(x)$, $y_2 = f(f(x))$, $y_3 = f(f(f(x)))$ and so on, then what is the value of y_{501}?

(a) – 1/x (b) (x + 1)/(x − 1) (c) 501x – 1 (d) (x − 1)/(x + 1)

18. What is the equation of the straight line which passes through the point of intersection of the straight lines 3x + 4y – 11 = 0 and x + y – 3 = 0 and is parallel to the line 2x + 5y = 0?

(a) 5x – 2y – 12 = 0 (b) 2x + 5y – 12 = 0 (c) 2x + 5y – 9 = 0 (d) 5x + 2y – 9 = 0

19. If a and b are integers such that $\log_2(a + b) + \log_2(a - b) = 3$, then how many different pairs (a, b) are possible?

(a) 0 (b) 1 (c) 2 (d) 3

20. A 3-digit natural number 'abc', where a, b and c are distinct digits, when increased by 33.33% becomes 'cab'. When 'cab' is increased by 33.33% it becomes 'bca'. How many such numbers are there?

(a) 0 (b) 1 (c) 2 (d) 5

21. The perimeter of a triangle is 8 cm and one of the sides is 3 cm. Find the other two sides if the area of the triangle is maximum.

(a) (5/2, 5/2) (b) (3/2, 3/2) (c) (3/2, 7/2) (d) (3/2, 5/2)

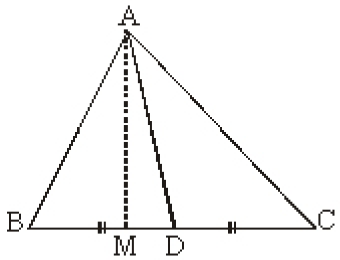
22. A horizontal pipe for carrying flood water has diameter of 1 m. When water in it is 10 cm deep, find the depth of the water surface.

(a) 30 cm (b) 60 cm (c) 50 cm (d) 70 cm

23. In a shooting competition a shooter has to hit any point on the target board in his last shot to win the tournament. His gun deviates by in left or right when he shoots. If he is standing 15 m away from the board and the direction of his gun is normal to the centre of the target board, what should be the diameter of the board so that he surely wins?

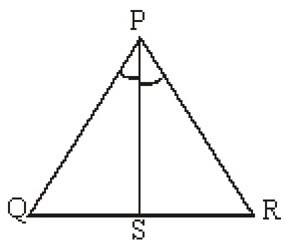
(a) 10 m (b) $10\sqrt{3}$ m (c) $11\sqrt{3}$ m (d) $15\sqrt{3}$ m

24. In figure, AD is a median of a triangle ABC and AM ⊥ BC . If AB = 4 cm, BC = 6 cm and AC = 8 cm then find AD.



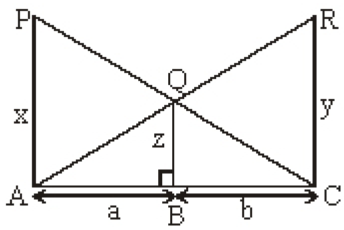
(a) 31 cm (b) √31 cm (c) 33 cm (d) √33 cm

25. In figure, PS is the bisector of ∠QPR of â–³ PQR . If PQ = 14 cm, PR = 12 cm and QS = 7 cm the find QR.



(a) 9 cm (b) 11 cm (c) 12 cm (d) 13 cm

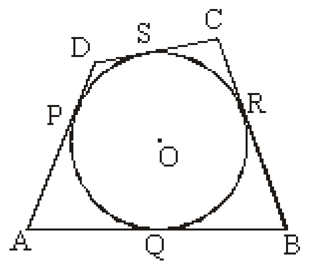
26. In the given Fig, if PA, QB and RC are each perpendicular to AC then



(a) $\frac{1}{x}-\frac{1}{y}=\frac{1}{z}$ (b) $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$

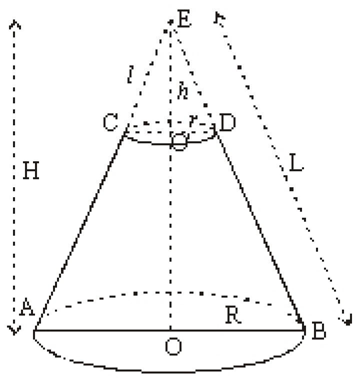
(a) $\frac{1}{x}-\frac{1}{z}=\frac{1}{y}$ (b) $\frac{1}{x}+\frac{1}{z}=\frac{1}{y}$

27. In the given figure, quadrilateral ABCD is circumscribed touching the circle at P, Q, R and S. If AP = 5 cm, BC = 7 cm, and CS = 3 cm, AB =?



(a) 10 cm (b) 8 cm (c) 12 cm (d) 9 cm

28. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface area of the remainder is 8/9 of the curved surface of the whole cone, find the ratio of the line-segment into which the one's altitude is divided by the plane.

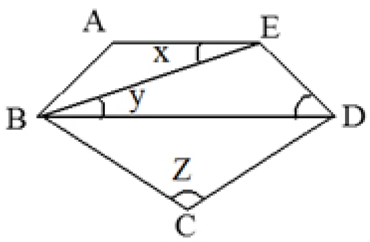


(a) 1:2 (b) 1:3 (c) 1:4 (d) 1:5

29. If two vertices of an equilateral triangle is (0 , 0), (3, v3)find the third vertex.

(a) (3 , √3) (b) (0 , 2√3) (c) (√3 , 3) (d) (√3 , 0)

30. Inthe figure ABCD is a regular pentagon. The measure of the angles marked y is



(a) 72° (b) 78° (c) 36° (d) 112°

31. If n is any positive integer greater than 1, then $(2^{3n} - 7n - 1)$ must be divisible by

(a) 9 (b) 25 (c) 36 (d) 49

32. The sum of all 4 digit numbers formed with the digits 1, 2, 4 and 6 is

(a) 86650 (b) 86660 (c) 86658 (d) 76650

33. Three gallons are drawn from a cask full of wine containing 27 gallons. The cask is then filled with water. Three gallons of mixture are again drawn and the cask is again filled with water. What is the ratio of water to wine now?

(a) 27/64 (b) 64/81 (c) 8/9 (d) None

34. An article is sold at a profit of 20%. If both the cost price and selling price are Rs. 100 less, the profit would be 4% more. Find the cost price.

(a) 500 (b) 600 (c) 560 (d) 660

35. 24 persons took a piece of work, which they can do in 18 days. After the work was done for some days by them all, 6 of them left and the work was carried to completion by the remaining persons. If the total period required to complete the work was 21 days. Find after how may days from the start of the work the 6 persons left.

(a) 6 (b) 7 (c) 9 (d) 18

36. Mickey and Donald set out on a morning walk every day at the same time from two opposite points. After passing each other, they finish their journey in 4 and 6 hours respectively. At what rate does Mickey walk if Donald walks at the rate of 2 kmph?

(a) 6 kmph (b) 8 kmph (c) 4 kmph (d) 2 kmph

37. There are 8 pairs of globes of different sizes. In how many ways can you choose one for the left hand and one for the right hand such that they are not of the same pair?

(a) 56 (b) 96 (c) 112 (d) 120

38. If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) gof (–2) = – 2 (b) gof (4) = 4 (c) gof (3) = 6 (d) gof (2) = 4

39. If $f(x) = \frac{1-x}{1+x}$, then which of the following is not the domain of $f^{-1}(x)$.

(a) $(-\infty , \infty)$ (b) $(-\infty , 1)$ (c) $(1,\infty)$ (d) $(1,\infty)$

40. Which of the following is an odd function?

(a) f(x) = cosx (b) $y = 2^{-x^2}$ (c) $y = 2^{x-x^4}$ (d) None

41. The ten numbers $x_1.x_2.x_3........x_{10}$ have a mean of 10 and a standard deviation of 3. Find the value of $\sum_{i=1}^{10}(x_i-12)^2$.

(a) 110 (b) 115 (c) 125 (d) 130

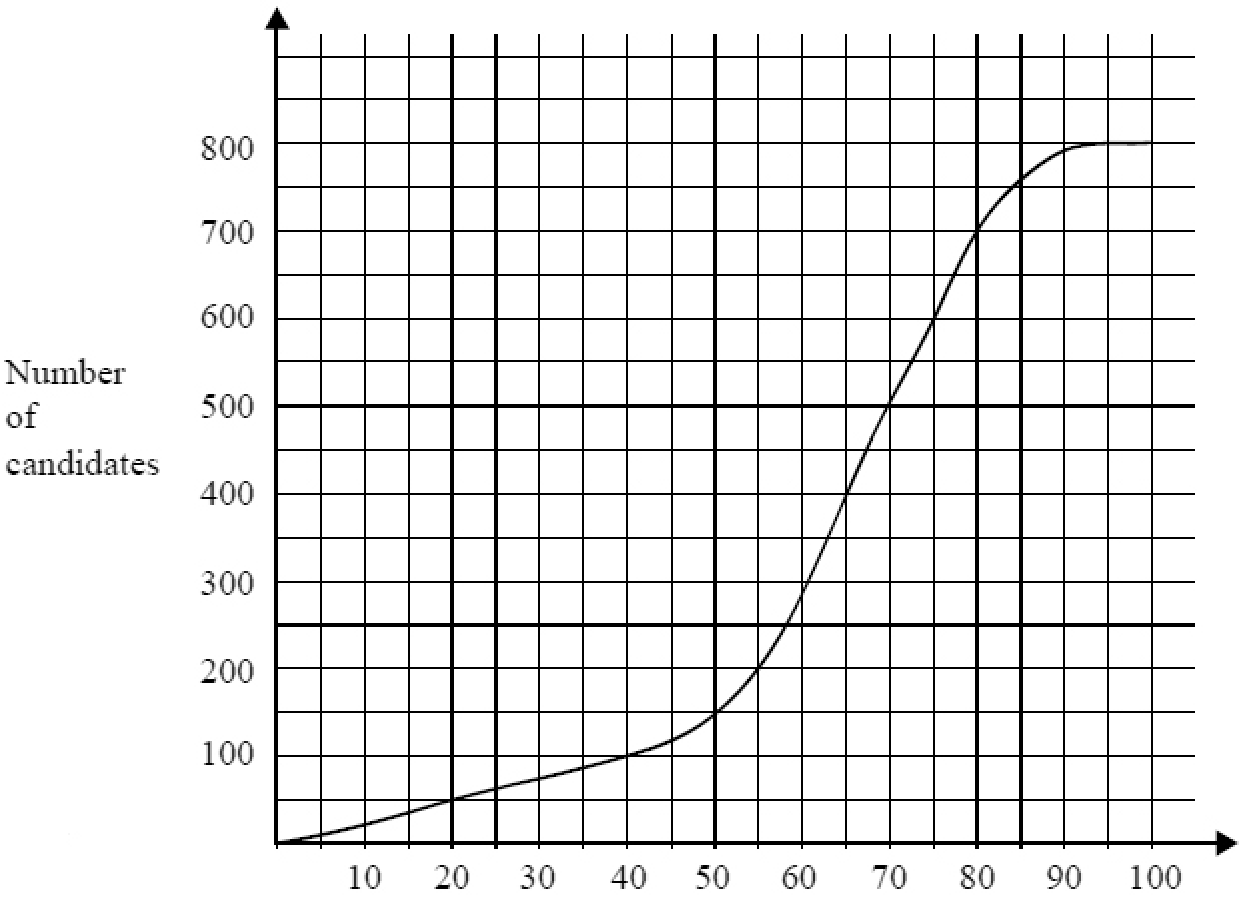
42. Jenny goes to school by bus every day. When it is not raining, the probability that the bus is late is 3/20 . When it is raining, the probability that the bus is late is 7/20 . The probability that it rains on a particular day is 9/20 . On one particular day the bus is late. Find the probability that it is not raining on that day.

(a) 9/11 (b) 11/32 (c) 11/26 (d) 11/12

43. If P(A) = 1/6 , P(B) = 1/3 , and P(A ∪ B) = , what is P(A′ / B′)?

(a) 5/6 (b) 6/7 (c) 7/8 (d) 8/9

44. A test marked out of 100 is written by 800 students. The cumulative frequency graph for the marks is given below.



The middle 50 % of test results lie between marks a and b, where a Find (a + b).

(a) 110 (b) 120 (c) 130 (d) 140

45. If the mean of the data 21, 25, 17, (x + 3), 19, (x – 4) is 18, then find the mode of the data.

(a) 14 (b) 15 (c) 16 (d) 17

46. There are 50 boxes in a factory. Their weights, w kg, are divided into 5 classes, as shown in the following table.

Class	Weight (kg)	Number of boxes
A	$9.5 \leq w < 18.5$	7
B	$18.5 \leq w < 27.5$	12
C	$27.5 \leq w < 36.5$	13
D	$36.5 \leq w < 45.5$	10
E	$45.5 \leq w < 54.5$	8

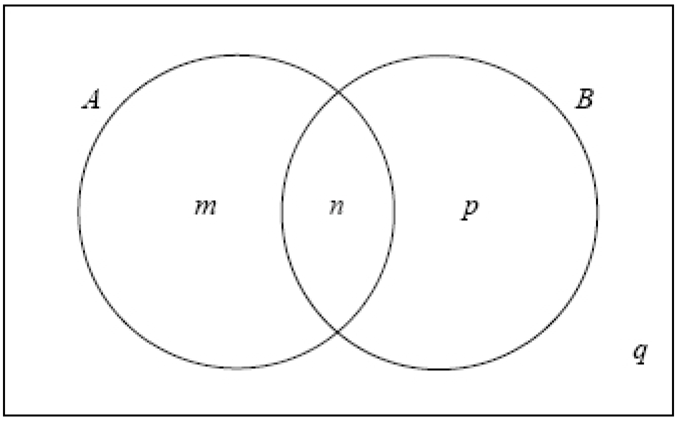
Find the estimated mean weight.

(a) 13 (b) 26 (c) 32 (d) 36

47. Consider the four numbers a, b, c, d with $a \leq b \leq c \leq d$, where a, b, c, d ε Z. The mean of the four numbers is 4, mode is 3, median is 3 and the range is 6, then Find the value of (a + b + c - d).

(a) 12 (b) 14 (c) 4 (d) 0

48. The Venn diagram shows events A and B where P(A) = 0.3, P(A∪B) = 0.6 and P(A∩B) = 0.1. Find P(B′).



(a) 0.1 (b) 0.3 (c) 0.6 (d) 0.9

49. Consider the independent events A and B. If P(B) = 2P(A), and P(A∪B) = 0.52,find P(B).

(a) 0.2 (b) 0.4 (c) 0.6 (d) 0.8

50. For independent events A_1, A_2, A_3,..........,A_n , $P(A_i)$ where i = 1, 2, . . ., n. Then the probability that none of the events will occur is

(a) n/n+1 (b) n-1/n+1 (c) 1/n+1 (d) 1-n/n+1

51. The number of ways in which a mixed double game can be arranged from amongst 9 married couples if no husband and wife play in the same game is

(a) 756 (b) 1296 (c) 1512 (d) 3024

52. P and Q are two points 100 km apart. A starts running from P towards Q at 10 km/hr. B starts running from Q at exactly the same time and in the same direction as that of A at 20 km/hr. After an hour, B turns back and changes his speed to 10 km/hr. After another hour, B again turns back and changes his speed to 20 km/hr. He keeps on changing his speed and direction in this manner till the time he meets A. After how much time will A and B meet for the first time?

(a) 30 hours (b) 18 hours (c) 10 hours (d) 20 hours

53. P and Q are two points 100 km apart. A starts running from P towards Q at 10 km/hr. B starts running from Q at exactly the same time and in the same direction as that of A at 20 km/hr. After an hour, B turns back and changes his speed to 10 km/hr. After another hour, B again turns back and changes his speed to 20 km/hr. He keeps on changing his speed and direction in this manner till the time he meets A. After how much time will A and B meet for the first time?

The question given below is followed by two statements, A and B. Mark the answer using the following instructions:

Mark (a) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Mark (b) if the question can be answered by using either statement alone.

Mark (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

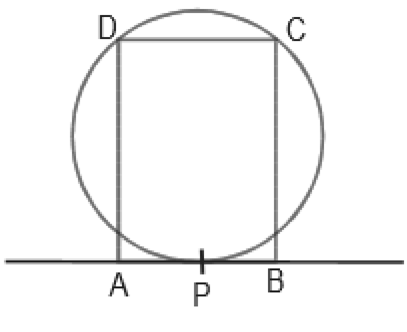
Mark (d) if the question cannot be answered even by using both the statements together.

Q. ABCD is a cyclic quadrilateral in which AB = 8 cm and BC = 15 cm. What is the area of the quadrilateral?

A. AD = CD
B. The length of the diameter of the circumcircle of triangle BCD is 17 cm.

54. A 3-digit natural number 'abc' , where a, b and c are distinct digits, when increased by 33.33% becomes 'cab'. When'cab' is increased by 33.33% it becomes 'bca'. How many such numbers are there?
(a) 0 (b) 1 (c) 2 (d) 5

55. In the figure given below, a tangent is drawn at point P on a circle of radius 1 cm. A and B are two points on the tangent and ABCD is a rectangle, where C and D are two points on the circumference of the circle. What is the approximate area (in cm^2) of the rectangle ABCD if 2AB = BC?



(a) 1.77 (b) 1.50 (c) 1.83 (d) 1.60

56. In how many ways can 18 identical balls be distributed among 3 identical boxes?
(a) 25 (b) 210 (c) 105 (d) 37

57. One hundred ml of alcohol is mixed with y ml of water. Forty ml of this alcohol-water mixture is added to 2y ml of another alcohol-water mixture whose alcohol concentration is 26%. If the percentage of water in the resultant mixture is 2y%, then what is the value of y?

(a) 30 (b) 40 (c) 20 (d) 25

58. If a and b are real numbers such that $a^{ab} = b$ and $a \neq b$, then what is the value of a^{b},– b?

(a) –1 (b) 0 (c) 1 (d) 2

59. A function f(x) is defined for all real values of x as f(x)= (x-1)/(x+1). If $y_1 = f(x)$, $y_2 = f(f(x))$, $y_3 = f(f(f(x)))$ and so on, then what is the value of y_{501}?

(a) -1/x (b) (x+1)/(x-1) (c) 501x– 1 (d)(x-1)/(x+1)

60. What is the equation of the straight line which passes through the point of intersection of the straight lines 3x + 4y– 11 = 0 and x + y– 3 = 0 and is parallel to the line 2x + 5y = 0?

(a) 5x– 2y– 12 = 0 (b) 2x + 5y– 12 = 0 (c) 2x + 5y– 9 = 0 (d) 5x + 2y– 9 = 0

61. If a and b are integers such that $\log_2 (a+b)+\log_2(a-b)=3$, then how many different pairs (a, b) are possible?

(a) 0 (b) 1 (c) 2 (d) 3

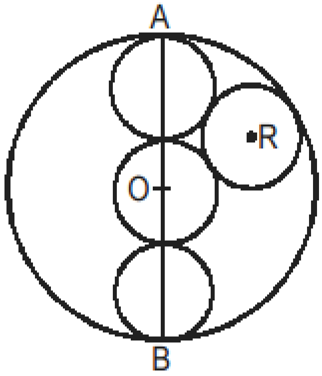
62. A cylindrical pipe of length 75 m, through which water flows at the rate of 54 km/hr, can fill 80% of a cuboidal tank of 118800 m3 capacity in 14 hours. What is the radius (in cm) of the cross section of the pipe?

(a) 20 (b) 25 (c) 50 (d) Cannot be determined

63. A large cube is formed by bringing together 729 smaller identical cubes. Each face of the larger cube is painted with red colour. How many smaller cubes are there none of whose faces is painted?

(a) 216 (b) 256 (c) 343 (d) None of these

64. In the figure given below, AB is the diameter of the larger circle while three smaller circles are drawn inside this circle such that their diameters are along AB. The radi**63.**us of each of these three circles is 1 cm and the length of AB is 6 cm. Another circle with center at R is drawn which touches the two smaller circles and the larger circle. What is the length of the radius (in cm) of this circle?



a) √3/2 (b)1/√2 (c) 1 (d) None of these

65. From the first 20 natural numbers how many Arithmetic Progressions of five terms can be formed such that the common difference is a factor of the fifth term?

(a) 16 (b) 22 (c) 25 (d) 26

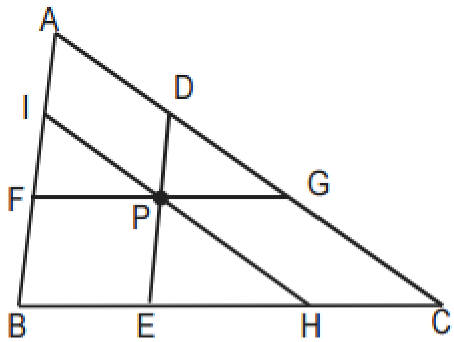
66. $5f(x)+4f\left(\frac{4x+5}{x-4}\right)=9(2x+1),$ where $x \in R$ and $x \neq 4.$ What is the value of f(7)?

(a) – 17 (b) – 8 (c) – 7 (d) None of these

67. There were 4 parcels all of whose weights were integers (in kg). The weights of all the possible pairs of parcels were noted down and amongst these the distinct values observed were 94 kg, 97 kg, 101 kg and 104 kg. Which of the following can be the weight of one of the parcels?

(a) 40 kg (b) 45 kg (c) 48 kg (d) 53 kg

68. In the figure given below, P is a point inside the triangle ABC. Line segments DE, FG and HI are drawn through P, parallel to the sides AB, BC and CA respectively. The areas of the three triangles DPG, FPI and EPH are 1, 9, and 25 respectively. What is the area of the triangle ABC? (All the areas are in sq cm).



(a) 81 (b) 144 (c) 16 (d) 64

69. Guppy has a watch that shows the date without the month and the year. By default, the watch displays 31 days in each month. Therefore, at the end of all the months with less than 31 days the date on the watch needs to be readjusted. On 10th March 2001 it showed the right date as '10'. What date would it show on 15th May 2002, if it is known that Guppy never readjusted his watch during this period?

(a) 23 (b) 7 (c) 8 (d) 22

70. Let P be a point on the side AB of a triangle ABC. Lines drawn parallel to PC, through the points A and B, meet BC and AC extended at X and Y respectively. The lengths of AX, BY and PC are
'a' units, 'b' units and 'c' units respectively. Then c will be equal to the half of

(a) Arithmetic Mean of a and b (b) Geometric Mean of a and b (c) Harmonic Mean of a and b (d) None of these

71. A game consisting of 50 rounds is played among P, Q and R as follows:
Two players play in each round and the player who loses in that round is replaced by the third player in the next round. If the only rounds in which P played against Q are the 3rd, 14th, 25th and 36th, then what can be the maximum number of games won by R?

(a) 40 (b) 42 (c) 41 (d) 36

72. A is the set of the first 100 natural numbers. What is the minimum number of elements that should be picked from A to ensure that atleast one pair of numbers whose difference is 10 is picked?

(a) 51 (b) 55 (c) 20 (d) 11

73. (X + 3)/3, (X + 8)/4, (X + 15)/5, (X + 24)/6 ...(X + 80)/10 is a sequence where X ≠ 1. What is the least value of X for which HCF (Numerator, Denominator) = 1 for each term of the given sequence?

(a) 17 (b) 13 (c) 11 (d) None of these

74. What is the number of non-negative integer solutions for the equation x2 – xy + y2 = x + y?

(a) 3 (b) 4 (c) 1 (d) None of these

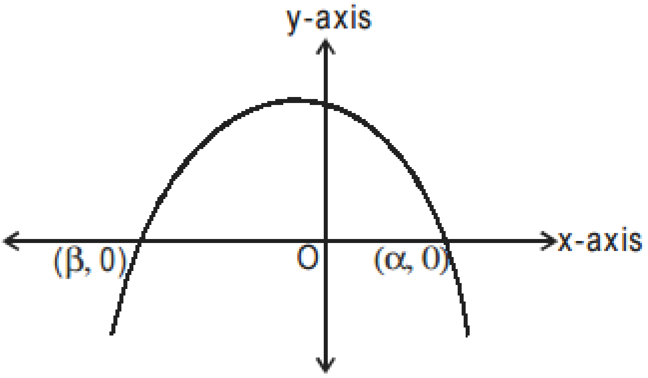
75. A sequence of non-negative integers is given such that t1 = 150 and n n - 2 n - 1 t = t – t for n > 2. For what value of t2 would the sequence have the maximum possible number of terms?

(a) 90 (b) 97 (c) 93 (d) 75

76. Anshul and Nitish run between point A and point B which are 6 km apart. Anshul starts at 10 a.m. from A, reaches B, and returns to A. Nitish starts at 10:30 a.m. from B, reaches A, and comes back to B. Their speeds are constant with Nitish's speed being twice that of Anshul's. While returning to their starting points they meet at a point which is exactly midway between A and B. When do they meet for the first time?

(a) 1 10 : 33 1/3a.m. (b) 2 10 : 37 2/3 a.m. (c) 10 : 33 a.m. (d) 2 10 : 33 2/3 a.m

77. The graph of $y = ax^2 + bx + c$ is shown below. If it is given that $|\alpha| < |\beta|$, then which of the following is true?



(a) a < 0, b < 0, c < 0 (b) a < 0, b > 0, c > 0

(c) a < 0, b < 0, c > 0 (d) a < b, b > 0, c < 0

78. A and B are moving along the circumference of a circle with speeds that are in the ratio 1 : K. They start simultaneously from a point P in the clockwise direction. They meet for the first time at a point Q which is at a distance of one-third the circumference from P, in the clockwise direction. K cannot be equal to

(a)1/4 (b)4/7 (c) 4 (d) None of these

79. In triangle PQR, PQ = PR = 10 cm. Points S, T and U lie on PQ, QR and PR respectively such that ST is parallel to PR and UT is parallel to PQ. What is the perimeter (in cm) of the quadrilateral PSTU?

(a) 18 (b) 20 (c) 24 (d) Data Insufficient

80. If 'x' is a real number then what is the number of solutions for the equation $\sqrt{(x^4 + 16)} = x^2 - 4$?

(a) 0 (b) 1 (c) 2 (d) 3

81. N! is completely divisible by 13^{52}. What is sum of the digits of the smallest such number N?

(a) 11 (b) 15 (c) 16 (d) 19

82. The lengths of the three edges of a cuboid are increased by a%, b% and c%. The volume increases by V%, where V is an integer. How many values can V take if a, b, c are real numbers and $10 \le a, b, c \le 20$?

(a) 11 (b) 39 (c) 41 (d) Cannot be determined

83. The question given below is followed by two statements, A and B. Mark the answer using the following instructions:

Mark (a) if the question can be answered by using either statement alone.

Mark (b) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Mark (c) if the question cannot be answered even by using both the statements together.

Mark (d) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Q. In a class of 200 students, the highest and the lowest scores in a test are 98 and 18 respectively. Is 50 the average score of the class in the test?

A. 100 students score above 50 and the remaining 100 students score below 50 in the test.

B. If the highest score and the lowest score in the test are excluded, the sum of the top 99 scores is exactly double of the sum of the bottom 99 scores.

84. What is the total number of ways of selecting twenty balls from an infinite number of blue, green and yellow balls?

(a) 3^{20} (b) 20^3 (c) 231 (d) 1771

85. In a class comprising 60 boys and some girls, the average age of boys is 14.8 years and that of girls is 14.1 years. If the average age of the class is 14.7 years, then how many girls are there in the class?

(a) 10 (b) 15 (c) 20 (d) 25

86. If m and n are positive integers such that$(m – n)^2 = 4mn/(m + n – 1)$, then how many pairs (m, n) are possible?

(a) 4 (b) 10 (c) 16 (d) Infinite

87. The lengths of the hour hand and the minute hand of a clock are 3.5 cm and 5.25 cm respectively. If the hour hand covers an area of 7.7 cm2, then find the approximate area (in cm^2) covered by the minute hand during the same time period.

(a) 17 (b) 158 (c) 260 (d) 208

88. In âˆ†ABC, M is the midpoint of AB and N is the midpoint of AC. CM and BN meet at point O and are perpendicular to each other. The length of AB is $2\sqrt{13}$cm and that of AC is $\sqrt{73}$ cm. What is the length of BC (in cm)?

(a) 17 (b) 19.25 (c) 8 (d) 5

89. There are 13 equidistant bus stops on a straight road. A bus running at 60 km/hr is some distance away from the 1st stop from where it will move towards the 13th stop. Two cars start running from the 6th stop in opposite directions with the same speed. If the bus meets one of the cars at the 1st stop and the other at the 13th stop, then find the speed of the cars.

(a) 10 km/hr (b) 20 km/hr (c) 30 km/hr (d) Cannot be determined

90. How many divisors of 25200 can be expressed in the form 4n + 3, where n is a whole number?

(a) 6 (b) 8 (c) 9 (d) None of these

91. The HCF of three natural numbers x, y and z is 13. If the sum of x, y and z is 117, then how many ordered triplets (x, y, z) exist?

(a) 28 (b) 27 (c) 54 (d) 55

92. n is a natural number such that ${}^{n}C_{4}={}^{n}C_{12}$. What is the remainder when n! is divided by n + 1?

(a) n – 1 (b) n – 2 (c) n (n) 0

93. What is the number of common tangents of the circles $x^2 + y^2 – 2x – 2y – 23 =$ and $x^2 + y^2 – 12x – 26y + 141 = 0$?

(a) 0 (b) 2 (c) 3 (d) 4

94. $U = 5(\log_2 x)^2 – 5(\log_2 x) – 8$, where x is a real number. If $x^U = 16$, find the value of x.

(a) 1 (b) 2 (c) 4 (d) 8

95. The digits of a 3-digit number in Base 4 get reversed when it is converted into Base 3. How many such numbers exist?

(a) 0 (b) 1 (c) 2 (d) 3

96. A = {3, 23, 43 ....603} and S is a subset of A. If the sum of no two elements of S is more than 606, then what can be the maximum possible number of elements in S?

(a) 15 (b) 14 (c) 17 (d) 16

97. The solution set for $|5x + 2| \leq 10$ is

(a) $8/5 \leq x \leq 12/5$ (b) $-12/5 \leq x \leq -8/5$ (c) $-8/5 \leq x \leq 12/5$ (d) $-12/5 \leq x \leq 8/5$

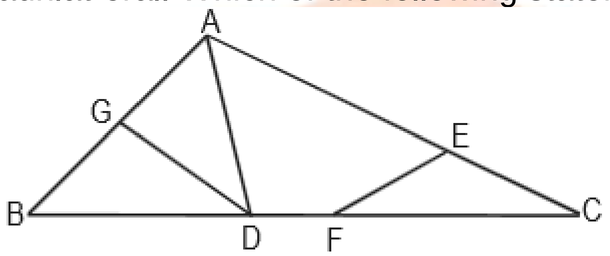
98. There are two Arithmetic Progressions A and B such that their n^{th} terms are given by $A_n = 101 + 3(n-1)$ and $B_n = 150 + (n-1)$, where n is the set of natural numbers. The first 50 terms of A and B are written alternately i.e. $A_1B_1A_2B_2.....A_{50}B_{50}$. What is the remainder when the number so formed is divided by 11?

(a) 0 (b) 1 (c) 9 (d) 10

99. How many 4-digit multiples of 3 can be formed using the digits 2 and 3 only?

(a) 4 (b) 6 (c) 5 (d) 7

100. In the figure given below, BG = GA = GD, AD = BD and EF = EC. Also, ADFE is a cyclic quadrilat-eral. Which of the following statements is/are definitely true?



(i) The orthocentre of triangle ABC lies at point A.

(ii)âˆ†GBD and âˆ†GDA are congruent.

(iii)AD is a median of triangle ABC

(iv) AD/EF = √2

(a) (i)and(iii) (b)(i),(ii)and(iii) (c) (ii),(iii)and(iv) (d)All four are true

QA_ANSWER KEY

2. (c)	3. (b)	4. (a)	5. (a)
6. (d)	7. (c)	8. (c)	9. (d)
10. (d).	11. (d)	12. (a)	13. (c)
14. (b)	15. (c)	16. (b)	17. (d)
18. (b)	19. (c)	20. (c)	21. (a)

22. (b)	23. (b)	24. (b)	25. (d)
26. (b)	27. (d)	28. (a)	29. (b)
30. (c)	31. (d)	32. (c)	33. (d)
34. (b)	35. (c)	36. (c)	37. (a)
38. (b)	39. (a)	40. (d)	41. (d)
42. (b)	43. (c)	44. (c)	45. (d)
46. (c)	47. (d)	48. (c)	49. (b)
50. (c)	51. (c)	52. (d)	53. (d)
54. (c)	55. (a)	56. (d)	57. (d)
58. (b)	59. (d)	60. (b)	61. (c)
62. (a)	63. (c)	64. (c)	65. (c)
66. (c)	67. (a)	68. (b)	69. (a)
70. (b)	71. (c)	72. (b)	73. (a)
74. (d)	75. (c)	76. (a)	77. (c)
78. (d)	79. (b)	80. (a)	81. (c)
82. (b)	83. (c)	84. (c)	85 (a)
86. (d)	87. (d)	88. (d)	89. (a)
90. (c)	91. (b)	92. (c)	93. (c)
94. (c)	95. (b)	96. (d)	97. (d)
98. (a)	99. (c)	100.(b)	

Detail Solutions

1. If $4^{x-5+y} = 2^{x+y} \times 2^{y} \times 2^{x-4} - 63$, find the sum of x and y

(a) 2 (b) 3 (c) 4 (d) 5

Ans: **â^µ** $4^{x-5+y} = 2^{x+y} \times 2^{y} \times 2^{x-4}$

=> $4^{(x+y)-5} = 2^{2(x+y)-4} - 63$

=> $4^{(x+y)-5} - 4^{(x+y)-2} = -63$

=> $4^{(x+y)-2} (4^{3} - 1) = -63$

+

=> $4^{(x+y)-2}$ (-63/64) = - 63

=> $4^{(x+y)-2} = 64$

=> $4^{(x+y)-2} = 4^3$

∴ x + y -2 = 3

∴ x + y = 5

2. Five apples and four oranges cost as much as three apples and seven oranges. Find out the ratio of the cost of one apple to the cost of one orange.

(a) 3:1 (b) 1:3 (c) 3:2 (d) 2:3

Ans: Let A stands for apples and O stands for oranges.

It is given that 5A + 4O = 7O+ 3A

=> 2A = 3O, ∴ A/O = 3/2

3. Given: $2x \geq 2^{x-1} + 2^{x-2} + \dots\dots$ *upto* 2^0 If x is an integer then find the value(s) of x.

(a) X=1 only (b) x=1, 2 only (c) x < 3 (d) x > 3

Ans: $\Rightarrow 2x \geq 2^{x-1}\left(\frac{1-2^{-x}}{1-2^{-1}}\right)$, (Terms of R.H.S. are in G.P.)

=> $2x \geq 2^x (1-2^x)$, => $2x \geq 2^x - 1$, which is true only for the values of x = 1 and 2.

4. If *4x - 8 + a = bx - 1* has an integer solution (*a,b*) then the values of *a* and *b* could be

(a) (4,1) (b) (2,4) (c) (4,2) (d) (6,2)

Ans: ∴ 4x - 8 + a =bx -1, => x(4 - b) = 7 - a

Now if x is an integer, then a = 4, b = 1 is a possible option.

5. If $x + y + z = 25$, (x,y,z >0), then the maximum value of $(x + y)(y + 3)(z)$ will be

(a) 1000 (b) 3125/27 (c) 3125 (d) ∞

Ans: âˆµ x + y + z = 25, => (x + 2) + (y + 3) + z = 25 + 5 = 30

$$\therefore \frac{(x+2)+(y+3)+z}{3} \geq \sqrt{(x+2)(y+3)z} \text{ (Since A.M. } \geq \text{ G.M.)}$$

$$\Rightarrow \frac{30}{3} \geq \sqrt{(x+2)(y+3)z}, \Rightarrow 10 \geq \sqrt{(x+2)(y+3)z}$$

=> 1000 $\geq$ (x + 2)(y + 3)(z) . Therefore, the maximum value of (x + 2)(y + 3)(z) is 1000.

6. Let x be a positive integer such that $x+7$ is divisible by 5. Find the smallest positive integer n where $n>2$ such that $x + n^2$ is divisible by 5.

(a) 3 (b) 4 (c) 5 (d) None

Ans: âˆµ x + 7 is divisible by 5. Hence x ends in either 3 or 8. Therefore for $x + n^2$ to be divisible by 5, n^2 has to end in either 2 or 7. Now since square of any number does not end in 2 or 7. Therefore, n cannot be found.

7. Find the even factors of 504.

(a) 12 (b) 15 (c) 18 (d) 24

Ans: 504 = 2^3 x 3^2 x 7^1. Let the powers of 2, 3 and 7 respectively be a, b and c.
Since an even divisor must have at least one factor of 2,

Therefore for even divisors, $1 \leq a \leq 3$, $0 \leq b \leq 2$ and $0 \leq c \leq 1$.

Therefore, number of even divisors is 3 x 3 x 2 = 18.

8. If $f(x) = 2x^3 - x + 2k$ and f (1) & f (2) are of opposite signs, then which of the following is true?

(a) $-7<k<1$ (b) $-5<k<1/2$ (c) $-7<k<-1/2$ (d) $-5<k<3/2$

Ans: âˆµ $f(x) = 2x^3 - x + 2k$, => f(1) = 2k + 1 and f(2) = 2k + 14
Now, **âˆµ** f(1) and f(2) are of opposite sign, ∴ (2k + 1)(2k + 14)< 0

Therefore, -7

Answer the questions 9 & 10 based on the following information:

H(a, b, c) = Greatest common divisor of a, b, c

L(a, b, c) = Least common multiple of a, b, c

A(a, b, c) = Average of a, b, c

Min(a, b, c) = Smallest value among a, b, c

Max(a, b, c) = Largest among a, b, c

9. If a, b, c are distinct positive real numbers then which of the following is true?

(a) H(a, b, c) × L(a, b, c) = abc *(b) H(a, b, c)>L(a, b, c)*

(c) H(a, b, c)>Min(a, b, c) *(d) H(a, b, c) < A(a, b, c)< L(a, b, c)*

Ans: (a) is true only for 2 numbers, i.e. GDC(a, b) X LCM(a, b) = a x b, GDC(a, b, c) X LCM(a,b, c) ≠ a x b x c.

(b) and (c) are false as average of 3 numbers can be greater than any one of them, i.e. average (a, b, c) > a is possible but GDC(a, b, c) has to be ≤a (assuming a < b and a < c).

Which leaves us with only choice (d), which is true.

10. If Max(a, b, c) = Min(a, b, c), then

(a) A(a, b, c) = H(a, b, c) (b) A(a, b, c) =L(a, b, c)

(c) A(a, b, c) = Min(a, b, c) (d) All of these

Ans: If max(a, b, c) = min(a, b, c), then a = b = c and with this all 3 are true

11. Each root of the equation $ax^3 – 7x^2 + cx + 231 = 0$ is an integer. One of the roots is −1/2 times the sum of the other two roots. What is the sum of all the possible values of a?

(a) 17 (b) – 7 (c) – 17 (d) None of these

Ans: Let the three roots of the equation be α , β γ , and .

Let us assume that α=−1/2 (β + γ) or β + γ = −2α

From the given equation we have:

α + β + γ = 7/a ...(i)

αβγ = − 231/a ...(ii)

Putting the value of β+γ in equation (i), we get

– 2α + α = 7/a or α = − 7/a

Putting the value of α in equation (ii), we get

βγ (−7/a) = − 231/a ⇒ βγ = 33.

The possible sets of values of α, β and γ are:

α	β/γ	γ/β
–17	1	33
–7	3	11
–17	–1	–33
7	–3	–11

As a = −7/α , for different values of α , the possible values of 'a' are –7/17, –7/17, 7/17 and 7/7

The required sum = 7{(−1/17) + (−1//7) + 1/17 + 1/7 } = 0

12. $M=\sqrt{3-\sqrt{5}+\sqrt{9-4\sqrt{5}}}$ and $N=\sqrt{\sqrt{7}-1-\sqrt{11-4\sqrt{7}}}$. What is the value of $\frac{M-N}{M+N}$?

(a) 0 (b) 1 (c) – 1 (d) None of these

Ans:

$$M = \sqrt{3-\sqrt{5}+\sqrt{9-4\sqrt{5}}}$$

$$= \sqrt{3-\sqrt{5}+\sqrt{\left((\sqrt{5})^2-2\times2\times\sqrt{5}+2^2\right)}}$$

$$= \sqrt{3-\sqrt{5}+\sqrt{(\sqrt{5}-2)^2}}$$

$$= \sqrt{3-\sqrt{5}+\sqrt{5}-2} = \sqrt{1} = 1$$

$$N = \sqrt{\sqrt{7}-1-\sqrt{11-4\sqrt{7}}}$$

$$= \sqrt{\sqrt{7}-1-\sqrt{\left(\sqrt{7}\right)^2-\left(2\times2\times\sqrt{7}\right)+(2)^2}}$$

$$= \sqrt{\sqrt{7}-1-\sqrt{\left(\sqrt{7}-2\right)^2}}$$

$$= \sqrt{\sqrt{7}-1-\sqrt{7}+2} = \sqrt{1} = 1$$

Hence, $\frac{M-N}{M+N} = \frac{1-1}{1+1} = 0.$

13. P + 1/Q = Q + 1/R = 1where P, Q and R are real numbers. What is the value of PQR + R + 1/P?

(a) –2 (b) – 1 (c) 0 (d) None of these

Ans:

$$P+\frac{1}{Q}=1 \Rightarrow \frac{1}{P}=\frac{1}{1-\frac{1}{Q}}=\frac{Q}{Q-1} \quad \text{...(i)}$$

$$Q+\frac{1}{R}=1 \Rightarrow R=\frac{1}{1-Q} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$R+\frac{1}{P}=\frac{1}{1-Q}-\frac{Q}{1-Q}=1 \quad \text{...(iii)}$$

$$\text{Also, } PQR=\left(\frac{Q-1}{Q}\right)Q\left(\frac{1}{1-Q}\right)=-1 \quad \text{...(iv)}$$

From (iii) and (iv), we get

$$PQR+R+\frac{1}{P}=1-1=0.$$

14. N = 70! × 69! × 68! × ..... 3! × 2! × 1! Which of the following represents the 147th digit from the right end of N?

(a) 2 (b) 0 (c) 5 (d) 7

Ans: We have to calculate the number of zeroes starting from the right end of the number N. The number of zeroes from:

1! to 4! = 0

5! to 9! = 1× 5 = 5

10! to 14! = 2 × 5 = 10

15! to 19! = 3 × 5 = 15

20! to 24! = 4 × 5 = 20

25! to 29! = 6 × 5 = 30

30! to 34! = 7 × 5 = 35

35! to 39! = 8 × 5 = 40

So we get 155 zeroes till 39! only. From this we can easily conclude that the 147th digit from the right end of N will be zero

15. A 3-digit natural number ‘abc’, where a, b and c are distinct digits, when increased by 33.33% becomes ‘cab’. When ‘cab’ is increased by 33.33% it becomes ‘bca’. How many such numbers are there?

(a) 0 (b) 1 (c) 2 (d) 5

Ans: abc × 1.33 = 4/3abc = cab ...(i)

cab × 1.33 = 4/3cab = 16/9abc = bca ...(ii)

From equation (ii), we can conclude that the resultant number is a multiple of 16 and the initial number is a multiple of 9. Hence, we can say that the resultant number should be a multiple of 16 as well as 9 i.e. a
multiple of 144.
There are two multiples of 144 which satisfy the condition i.e. 432 and 864.
Thus the number ‘abc’ could be either 243 or 486.

16. If a and b are real numbers such that $a_a b$ and a ≠ b, then what is the value of $a^b - b$?

(a) – 1 (b) 0 (c) 1 (d) 2

Ans:

It is given that $a^{a^b} = b$

Putting the value of b in left-hand side, we get

$a^{a^{a^{a^b}}} = b$

On repeating the same step n times, we get

$a^{a^{a^{\cdot^{\cdot^{\cdot^b}}}}} = b$

When n tends to infinity, we get

$a^{a^{a^{\cdot^{\cdot^{\cdot^b}}}}} = a^b = b$

Hence $a^b - b = 0$.

Alternate Method:

$a^2 = 2$, then $a = \sqrt{2}$

a_2a 2, then $a = \sqrt{3}$

Similarly, if $a_ab = b$, then $a = {}^{a}\sqrt{b} = (b)^{1/b}$

Hence, $a^b - b = 0$.

17: . A function f(x) is defined for all real values of x as x 1 $f(x) = (x - 1)/(x + 1)$. If $y_1 = f(x)$, $y_2 = f(f(x))$, $y_3 = f(f(f(x)))$ and so on, then what is the value of y_{501}?

(a) $-1/x$ (b) $(x + 1)/(x - 1)$ (c) $501x - 1$ (d) $(x - 1)/(x + 1)$

Ans: $y = (x - 1)/(x + 1)$

$y_2 = f(y_1) = -1/x$

$y_3 = f(y_2) = -(x + 1)/(x - 1)$

$y_4 = f(y_3) = x$

$y_5 = f(y_4) = (x - 1)/(x + 1)$

It can be concluded that the given function has the cyclicity of 4 or $y_n = y_{n+4k}$, where k is a whole number.

Hence, $y_{501} = y_1 = (x - 1)/(x + 1)$.

18. What is the equation of the straight line which passes through the point of intersection of the straight lines $3x + 4y - 11 = 0$ and $x + y - 3 = 0$ and is parallel to the line $2x + 5y = 0$?

(a) $5x - 2y - 12 = 0$ (b) $2x + 5y - 12 = 0$ (c) $2x + 5y - 9 = 0$ (d) $5x + 2y - 9 = 0$

Ans: Solving the two linear equations $3x + 4y - 11 = 0$ and $x + y - 3 = 0$, we get $x = 1$ and $y = 2$.

Hence, the two lines intersect at the point (1, 2).

Any line which is parallel to $2x + 5y = 0$ should be of the form $2x + 5y – k = 0$ …(i)

where k is a real number.

Putting $x = 1$ and $y = 2$ in (i), we get $k = 12$.

Hence, the equation of the straight line will be $2x + 5y – 12 = 0$.

19. If a and b are integers such that $\log_2(a + b) + \log_2(a - b) = 3$, then how many different pairs (a, b) are possible?

(a) 0 (b) 1 (c) 2 (d) 3

Ans: $\log_2(a + b) + \log_2(a - b) = 3$

$\Rightarrow \log_2(a + b)(a – b) = 3$

$\Rightarrow \log_2(a^2 – b^2) = 3$

$\Rightarrow a^2 – b^2 = 8$

Solving the above equation for integer values of a and b, we get (a, b) ≡ (3, 1) or (3, –1).

Note: ‘a – b’ must be greater than zero

20. A 3-digit natural number ‘abc’, where a, b and c are distinct digits, when increased by 33.33% becomes ‘cab’. When ‘cab’ is increased by 33.33% it becomes ‘bca’. How many such numbers are there?

(a) 0 (b) 1 (c) 2 (d) 5

Ans: abc × 1.33 = 4/3abc = cab ...(i)

cab × 1.33 = 4/3cab = 16/9abc = bca ...(ii)

From equation (ii), we can conclude that the resultant number is a multiple of 16 and the initial number is a multiple of 9. Hence, we can say that the resultant number should be a multiple of 16 as well as 9 i.e. a
multiple of 144.

There are two multiples of 144 which satisfy the condition i.e. 432 and 864.

Thus the number ‘abc’ could be either 243 or 486.

21. The perimeter of a triangle is 8 cm and one of the sides is 3 cm. Find the other two sides if the area of the triangle is maximum.

(a) (5/2, 5/2) (b) (3/2, 3/2) (c) (3/2, 7/2) (d) (3/2, 5/2)

Ans: Let one side of triangle be a, then other will be (5 - a).

$s = \frac{a + b + c}{2} = 8/2 = 4$

$$\therefore Area = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{4(4-3)(4-a)(4-5+a)}$$

$$\therefore Area = \sqrt{4\times 1\times (4-a)\times (a-1)}, \Rightarrow A^2 = -4a^2 + 20a - 16$$

$$\therefore Area = -4(a^2 - 5a + 4) = -4\left(a - \frac{5}{2}\right)^2 + 9$$

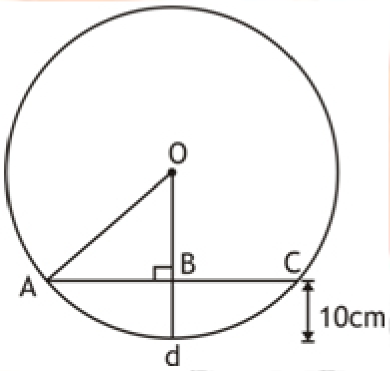
∴ For maximum area, a - 5/2 = 0, => a = 5/2

∴ Other possible side are 5/2 and 5/2.

22. A horizontal pipe for carrying flood water has diameter of 1 m. When water in it is 10 cm deep, find the depth of the water surface.

(a) 30 cm (b) 60 cm (c) 50 cm (d) 70 cm

Ans: Since the diameter = 1 m = 100 cm, ∴ Radius= 4 cm.



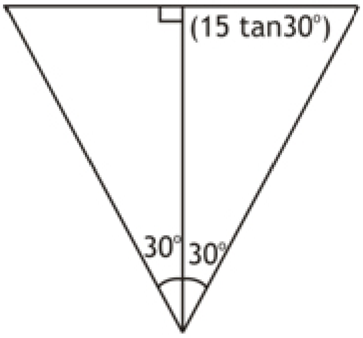
=> $50^2 = AB^2 + 40^2$, ∴ AB = 30 , ∴ BC = 30 cm Since water is 10 cm deep, OB = 40 cm, ∴ In ΔBAO, $OA^2 = AB^2 + OB^2$

∴ AC = AB + BC = 60 cm

23. In a shooting competition a shooter has to hit any point on the target board in his last shot to win the tournament. His gun deviates by in left or right when he shoots. If he is standing 15 m away from the board and the direction of his gun is normal to the centre of the target board, what should be the diameter of the board so that he surely wins?

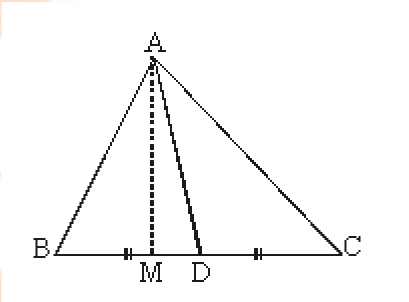
(a) 10 m (b) 10√3 m (c) 11√3 m (d) 15√3 m

Ans:



Required diameter of the target = 2 x tan 30° x 15 = 30 x 1/√3 = 10√3m

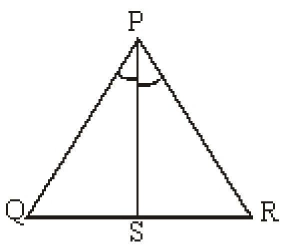
24. In figure, AD is a median of a triangle ABC and . If AB = 4 cm, BC = 6 cm and AC = 8 cm then find AD.



(a) 31 cm (b) √31 cm (c) 33 cm (d) √33 cm

Ans: **âˆµ** $AB^2 + AC^2 = 2AD^2 + 1/2\ BC^2$, ∴ $4^2 + 8^2 = 2AD^2 + (1/2)\ 6^2$, => $AD^2 = 62/2 = 31$
∴ AD = √31

25. In figure, PS is the bisector of ∠QPR of â–³ PQR . If PQ = 14 cm, PR = 12 cm and QS = 7 cm the find QR.

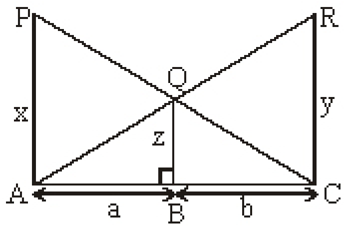


(a) 9 cm (b) 11 cm (c) 12 cm (d) 13 cm

Ans: Since PS is the bisector of ∠P, ∴ PQ/PR = QS/RS , => 14/12 = 7/RS, ∴ RS = 6,

∴ QR = QS + RS = 13 cm

26. In the given Fig, if PA, QB and RC are each perpendicular to AC then



(a) $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ (b) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

(a) $\frac{1}{x} - \frac{1}{z} = \frac{1}{y}$ (b) $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

Ans:

$\because \Delta CBQ \sim \Delta CAP$ (By AA criterion of similarity)

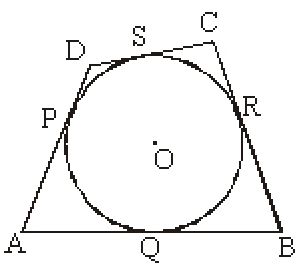
$\therefore \frac{BC}{AC} = \frac{BQ}{AP}, \Rightarrow \frac{b}{a+b} = \frac{z}{x}$ .... (i)

Similarly, , $\because \Delta ABQ \sim \Delta ACR \therefore \frac{AB}{AC} = \frac{BQ}{CR}, \Rightarrow \frac{a}{a+b} = \frac{z}{y}$ ...... (ii)

Adding (i) and (ii), $\frac{b}{a+b} + \frac{a}{a+b} = \frac{z}{x} + \frac{z}{y}, \therefore \frac{a+b}{a+b} = z\left(\frac{1}{x} + \frac{1}{y}\right)$

$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y}$

27. In the given figure, quadrilateral ABCD is circumscribed touching the circle at P, Q, R and S. If AP = 5 cm, BC = 7 cm, and CS = 3 cm, AB =?



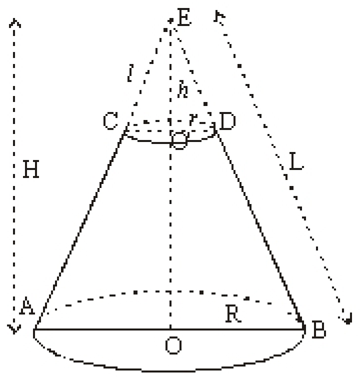
(a) 10 cm (b) 8 cm (c) 12 cm (d) 9 cm

Ans: Since AP = 5 cm, ∴ AQ = 5 cm (The lengths of tangents drawn from an external point to a circle are equal.

Similarly, CR = CS = 3 cm, ∴ BR = BC – CR = 4 cm. BQ = 4 cm

Now, AB = AQ + BQ = 5 cm + 4 cm = 9 cm

28. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface area of the remainder is 8/9 of the curved surface of the whole cone, find the ratio of the line-segment into which the one's altitude is divided by the plane.



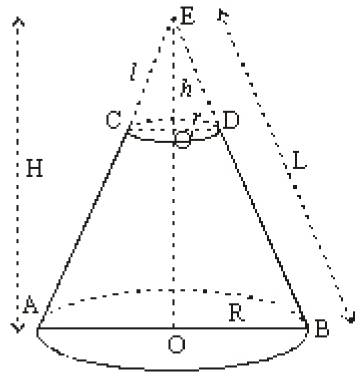
(a) 1:2 (b) 1:3 (c) 1:4 (d) 1:5

Ans: Let EAB be a hollow cone of height H, slant height L and base radius R. Suppose this cone is cut by a plane parallel to the base such that Oï,¢ is the centre of the circular section of the cone. Let h be the height, l be the slant height and r be the base radius of the smaller cone ECD in figure.

Now, â–³EO'C ~ â–³EOA

∴ EO'/EO = O'C/OA = EC/EA

=> h/H = r/R = l/L .....(i)



But it is given that:Curved surface area of the frustum ABCD = 8/9 x curved surface area of the cone EAB.

=> π (R + r) (L - l) = 8/9 x πRL

$$\Rightarrow (R+r)(L-l) = \frac{8}{9}\times RL$$

$$\Rightarrow \left(\frac{R+r}{R}\right)\left(\frac{L-l}{L}\right) = \frac{8}{9}, \Rightarrow \left(1+\frac{r}{R}\right)\left(1-\frac{l}{L}\right) = \frac{8}{9}$$

$$\Rightarrow \left(1+\frac{r}{R}\right)\left(1-\frac{h}{H}\right) = \frac{8}{9} \text{ ...... [From (i)]}$$

$$\Rightarrow \left(1-\frac{h^2}{H^2}\right) = \frac{8}{9}, \therefore \frac{h^2}{H^2} = \frac{1}{9},$$

$$\therefore \frac{h}{H} = \frac{1}{3}, \Rightarrow h = \frac{H}{3}$$

Hence, required ratio = $\frac{h}{H-h} = \frac{H/3}{H-H/3} = \frac{1}{2}$ [From (ii)]

29. If two vertices of an equilateral triangle is (0 , 0), (3, v3)find the third vertex.

(a) (3 , √3) (b) (0 , 2√3) (c) (√3 , 3) (d) (√3 , 0)

Ans: O(0,0) and A(3 , √3) be the given points and let B(x, y) be the third vertex of equilateral â–³OAB . Then, OA = OB = AB

=>$OA^2 = OB^2 = AB^2$

We have, $OA^2 = (3 – 0)^2 +$ = 9 + 3 = 12

$OB^2 = x^2 + y^2$

And $AB^2 = (x – 3)^2 + (y - \sqrt{3})^2$

=> $AB^2 = x^2 – 6x + 9 + y^2 - 2\sqrt{3}y + 3$

=> AB2 = $x^2 + y^2 – 6x - 2\sqrt{3}y + 12$

∴ $OA^2 = OB^2 = AB^2$

=> $OA^2 = OB^2$ and $OB^2 = AB^2$

=> 12 = $x^2 + y^2$ and $x^2 + y^2 = x^2 + y^2 - 6x - 2\sqrt{3}y + 12$

=> $x^2 + y^2 = 12$ and => $6x + 2\sqrt{3}y = 12$

=> $x^2 + y^2 = 12$ and $3x + \sqrt{3}y = 6$

$$\Rightarrow \quad x^2 + \left(\frac{6-3x}{\sqrt{3}}\right)^2 = 12 \qquad \left[\begin{array}{l} \because 3x + \sqrt{3}y = 6 \\ \therefore \qquad y = \frac{6-3x}{\sqrt{3}} \end{array}\right]$$

$$\Rightarrow \quad x^2 + \frac{(6-3x)^2}{3} = 12$$

$$\Rightarrow \quad \frac{3x^2 + (6-3x)^2}{3} = 12$$

=> $3x^2 + (6 - 3x)^2 = 36$

=> $3x^2 + 36 - 36x + 9x^2 = 36$

=> $12x^2 - 36x = 0$

∴ x = 0 and x = 3

∴ x = 0

=> $\sqrt{3}y = 6$

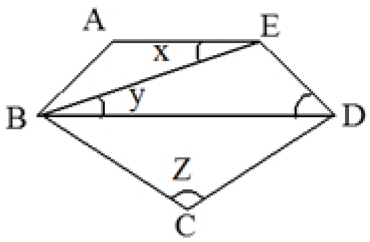
$$\Rightarrow \quad y = \frac{6}{\sqrt{3}} = \frac{2 \times 3}{\sqrt{3}} = \frac{2 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 2\sqrt{3}$$ [Putting x = 0 in $3x + \sqrt{3}y = 0$]

and x = 3

$$\Rightarrow \quad 9 + \sqrt{3}y = 6 \; \therefore \; y = \frac{6-9}{\sqrt{3}} = -\sqrt{3}$$ [Putting x = 3 in $3x + \sqrt{3}y = 6$]

Hence, the coordinates of the third vertex B are (0, 2√3) or (3, -√3)

30. In the figure ABCD is a regular pentagon. The measure of the angles marked y is



(a) 72° (b) 78° (c) 36° (d) 112°

Ans: In a regular pentagon, each interior angle = 108°

In triangle ABE = ∠A = 108°,

â^µ AB = AE, ∴ ∠ABE + ∠AEB = 180° – 108 = 72°

Since they are equal, ∠x = = 36°

âˆµ AE is parallel to BD, ∴ x = y = 36°

31. If n is any positive integer greater than 1, then (2^{3n} - 7n - 1) must be divisible by

(a) 9 (b) 25 (c) 36 (d) 49

Ans: Using binomial theorem,

$2^{3n} = 8^n = (1+7)^n = C(n,0) + C(n,1) \times 7 + C(n,2) \times 7^2 + ...+ C(n,n) \times 7^n$

$\Rightarrow 2^{3n} = 1 + 7n + 7^2 \times [C(n,2) + ...+ 7^{n-2}]$

$\Rightarrow 2^{3n} - 7n - 1 = 49 \times I$, where I = Integers

∴ (2^{3n} - 7n - 1) is divisible by 49

32. The sum of all 4 digit numbers formed with the digits 1, 2, 4 and 6 is

(a) 86650 (b) 86660 (c) 86658 (d) 76650

Ans: Required Sum = $(a_1 + a_2 + a_3 +........+ a_n)$ x(n-1)! x (111......n times)

= (1 + 2 + 4 + 6) x (4-1)! x(1111) = 13 × 6 × 1111 = 86658

33. Three gallons are drawn from a cask full of wine containing 27 gallons. The cask is then filled with water. Three gallons of mixture are again drawn and the cask is again filled with water. What is the ratio of water to wine now?

(a) 27/64 (b) 64/81 (c) 8/9 (d) None

Ans:

According to formula, $\frac{\text{Wine Left}}{\text{Total Capacity}} = \left(\frac{c-d}{c}\right)^n$

Where, d = mixture drawn at a time, c = capacity, n = number of operations

$\therefore \frac{\text{Wine Left}}{\text{Total Capacity}} = \left(\frac{27-3}{27}\right)^2 = \frac{64}{81}$

Water: Wine = (81 – 64): 64 = 17 : 64.

34. An article is sold at a profit of 20%. If both the cost price and selling price are Rs. 100 less, the profit would be 4% more. Find the cost price.

(a) 500 (b) 600 (c) 560 (d) 660

Ans: Suppose the cost price of that article is x. The selling price = 120 % of x = 120x/100.

New cost price = (x-100). New selling price = (120x/100) - 100

New profit = $\left(\frac{120x}{100} - 100\right) - (x-100) = \frac{120x}{100} - x = \frac{20x}{100}$.

$\therefore$ *New* **percentage profit** $= \left(\frac{20x/100}{x-100}\right) \times 100\% = \frac{20x}{x-100}\%$.

As new profit % = 24, $\therefore \frac{20x}{x-100} = 24$, $\therefore x = 600$

35. 24 persons took a piece of work, which they can do in 18 days. After the work was done for some days by them all, 6 of them left and the work was carried to completion by the remaining persons. If the total period required to complete the work was 21 days. Find after how may days from the start of the work the 6 persons left.

(a) 6 (b) 7 (c) 9 (d) 18

Ans: Let us suppose that 6 persons left the work after x days.

Then work done by 24 men in 18 days = work done by 24 men in x days + work done by (24 - 6) men in (21 - x) days.

=> 24 x 18 = 24x + 18(21-x), => 432 = 24x + 378 – 18x, => x = 9

Hence 6 persons left the work after 9 days.

36. Mickey and Donald set out on a morning walk every day at the same time from two opposite points. After passing each other, they finish their journey in 4 and 6 hours respectively. At what rate does Mickey walk if Donald walks at the rate of 2 kmph?

(a) 6 kmph (b) 8 kmph (c) 4 kmph (d) 2 kmph

Ans:

$$\frac{\text{Speed of first person}}{\text{Speed of second person}} = \sqrt{\frac{\text{Time taken by second person after passing the first}}{\text{Time taken by first person after passing the second}}}$$

$$\therefore \frac{\textbf{Micky's Speed}}{\textbf{2 kmph}} = \sqrt{\frac{16}{4}}, \therefore \textbf{Micky's Speed} = 4kmph$$

37. There are 8 pairs of globes of different sizes. In how many ways can you choose one for the left hand and one for the right hand such that they are not of the same pair?

(a) 56 (b) 96 (c) 112 (d) 120

Ans: Total number ways of selection of globes = C(8,1) x C(7,1) = 8 x 7 = 56

38. If f(x) = x^2 and g(x) = √x then

(a) gof (–2) = – 2 (b) gof (4) = 4 (c) gof (3) = 6 (d) gof (2) = 4

Ans: By definition, gof(x) = g(f(x)) = g(x^2) = |x|.

Therefore, gof(–2) = |– 2| = 2, gof(4) = 4, gof(3) = 3 and gof(2) = 2,

Therefore, all the answers except (b) are incorrect.

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39. If f(x) = $\frac{1-x}{1+x}$, then which of the following is not the domain of $f^{-1}(x)$.

(a) (-∞ , ∞) (b) (-∞ , 1) (c) (1,∞) (d) (1,∞)

Ans: Putting y = f(x) and solving for x, we have y + yx = 1 – x,

$$\Rightarrow x(1+y) = 1-y, \Rightarrow x = \frac{1-y}{1+y}, \therefore f^{-1}(x) = \frac{1-x}{1+x}$$

Therefore, domain of $f^{-1}(x)$ = R ~ {–1}.This is equivalent to saying that the domain of $f^{-1}(x)$ contains (–∞, –1), (1, ∞) and (0, ∞).

∴ Correct answer is (a).

40. Which of the following is an odd function?

(a) f(x) = cosx (b) $y = 2^{-x^2}$ (c) $y = 2^{x-x^4}$ (d) None

Ans: cosx is an even function since cos(–x) = cosx. Similarly, 2^{-x^2} is an even function. 2^{x-x^4}
But is neither even nor odd.

∴ (d) is the correct answer.

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41. The ten numbers $x_1.x_2.x_3........x_{10}$ have a mean of 10 and a standard deviation of 3. Find the value of $\sum_{i=1}^{10}(x_i-12)^2$.

(a) 110 (b) 115 (c) 125 (d) 130

Ans:

$$\sum_{i=1}^{10}(x_i-12)^2=\sum_{i=1}^{10}x_i^2-24\sum_{i=1}^{10}x_i+144\sum_{i=1}^{10}1$$

$$\bar{x}=10\Rightarrow\sum_{i=1}^{10}x_1=100$$

$$\sigma_x=3,\ \frac{\sum_{i=1}^{10}x_i^2}{10}-\bar{x}^2=9$$

$$\Rightarrow\sum_{i=1}^{10}x_i^2=10(9+100)$$

$$\sum_{i=1}^{10}(x_i-12)^2=1090-2400+1440=130$$

42. Jenny goes to school by bus every day. When it is not raining, the probability that the bus is late is 3/20 . When it is raining, the probability that the bus is late is 7/20 . The probability that it rains on a particular day is 9/20 . On one particular day the bus is late. Find the probability that it is not raining on that day.

(a) 9/11 (b) 11/32 (c) 11/26 (d) 11/12

Ans: P(R' ∩ L) = 11/20 x 3/20

P(L) = 9/20 x 7/20 + 11/20 x 3/20

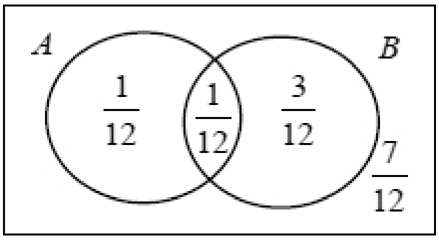
P(R′|L) = P(R' ∩ L)/P(L)

= 33/96 (= 11/32)

43. If P(A) = 1/6 , P(B) = 1/3 , and P(A ∪ B) = , what is P(A′ / B′)?

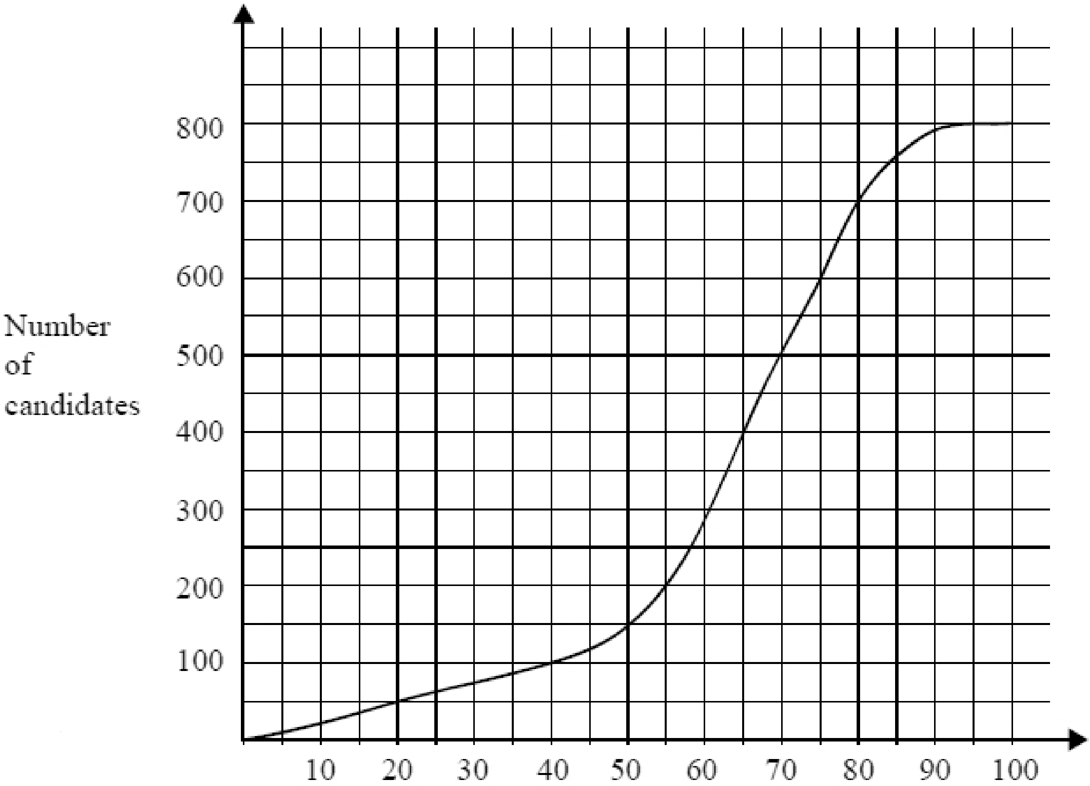
(a) 5/6 (b) 6/7 (c) 7/8 (d) 8/9

Ans: P (A ∩ B) = P (A) + P (B) – P (A ∪ B) = 2/12 + 4/12 - 5/12 = 1/12



$$\therefore P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{\frac{7}{12}}{\frac{8}{12}} = \frac{7}{8}$$

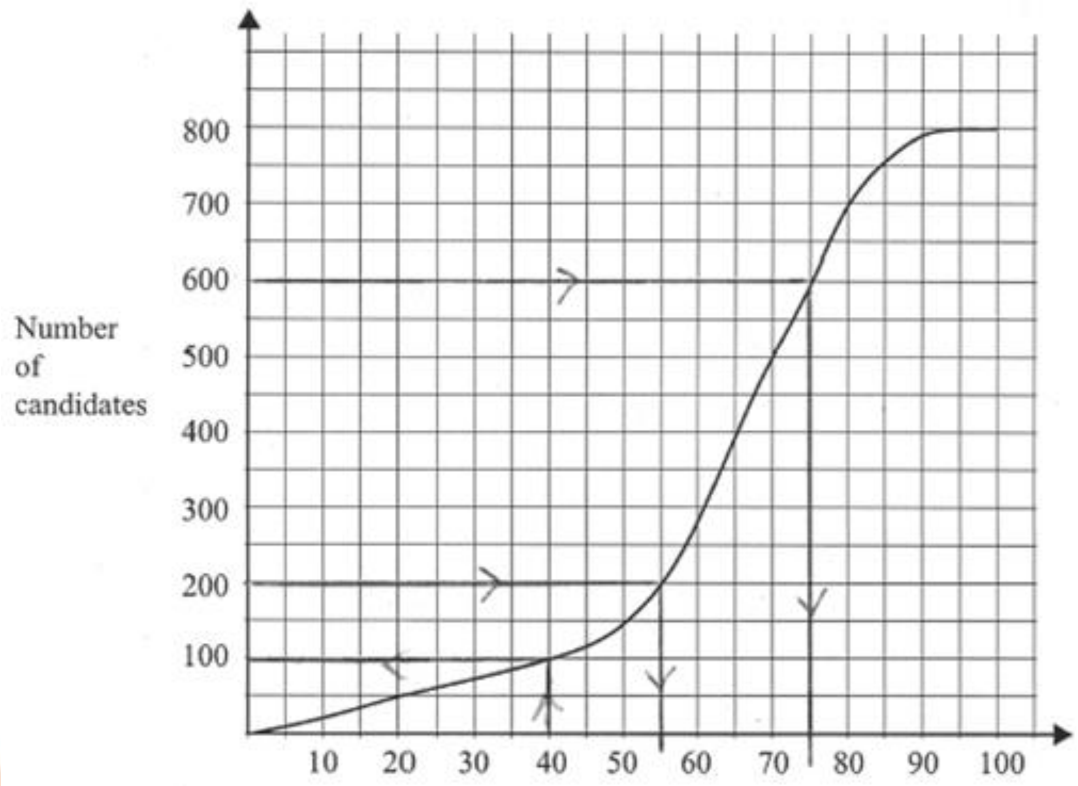
44. A test marked out of 100 is written by 800 students. The cumulative frequency graph for the marks is given below.



The middle 50 % of test results lie between marks a and b, where a

(a) 110 (b) 120 (c) 130 (d) 140

Ans:



Identifying 200 **and** 600 lines on graph, we get a = 55, b = 75.

∴ a + b = 55 + 75 = 130

45. If the mean of the data 21, 25, 17, (x + 3), 19, (x – 4) is 18, then find the mode of the data.

(a) 14 (b) 15 (c) 16 (d) 17

Ans: Given data: 21, 25, 17, (x + 3), 19, (x – 4), 3

Here, number of observations = 7

Mean =18,

∴ $$\frac{21+15+17+(x+3)+19+3}{7}=18$$

=> 126 = 24 + 2x

=> 2x = 126 – 84, x = 21

Now putting x = 21, the given data will be 21, 25, 17, 24, 19, 17, 3

Since 17 has the maximum frequency i.e. 2

∴ Mode of given data = 17

46. There are 50 boxes in a factory. Their weights, w kg, are divided into 5 classes, as shown in the following table.

Class	Weight (kg)	Number of boxes
A	$9.5 \le w < 18.5$	7
B	$18.5 \le w < 27.5$	12
C	$27.5 \le w < 36.5$	13
D	$36.5 \le w < 45.5$	10
E	$45.5 \le w < 54.5$	8

Find the estimated mean weight.

(a) 13 (b) 26 (c) 32 (d) 36

Ans: Correct mid interval values are 14, 23, 32, 41, 50

$$\therefore \bar{w} = \frac{7 \times 14 + 12 \times 23 + 13 \times 32 + 23 \times 41 + 8 \times 50}{50} = 32$$

47. Consider the four numbers a, b, c, d with a ≤ b ≤ c ≤ d, where a, b, c, dÎ Z. The mean of the four numbers is 4, mode is 3, median is 3 and the range is 6, then Find the value of (a + b + c - d).

(a) 12 (b) 14 (c) 4 (d) 0

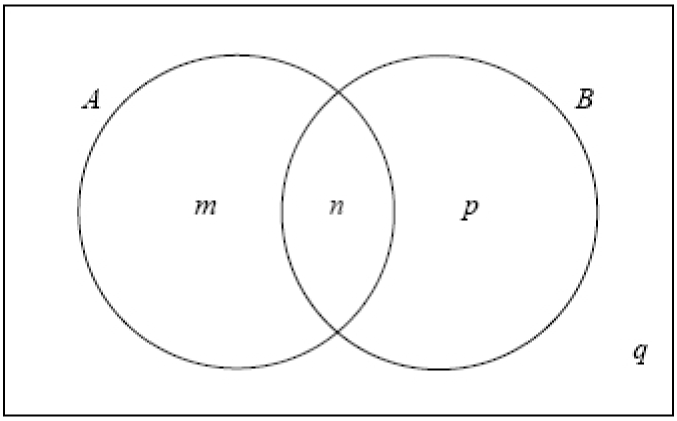
Ans: Median of a, b, c and d = 3, b = 3, c = 3

Using mean = $\frac{a + b + c + d}{2}$, we get a + d = 10

Now, using range = d – a = 6, d = 8 and a = 2

∴ a + b + c – d = 0

48. The Venn diagram shows events A and B where P(A) = 0.3, P(AUB) = 0.6 and P(A∩B) = 0.1. Find P(B′).



(a) 0.1 (b) 0.3 (c) 0.6 (d) 0.9

Ans: n = 0.1, m = 0.3 – 0.1 = 0.2,

âˆµ P (B) = P (AUB) + P (A∩B) - P (A)

=> P (B) = 0.6 + 0.1 - 0.3 = 0.4,

∴ P (B′) =1– P (B) = 1- 0.4 = 0.6

49. Consider the independent events A and B. IfP(B) = 2P(A), and P(AUB) = 0.52,find P(B).

(a) 0.2 (b) 0.4 (c) 0.6 (d) 0.8

Ans: For independent events A and B, P(A∩B) = P(A) × P(B)

âˆµ P(B) = 2P(A)

∴ P(AUB) = P(A) + P(B) – P(A∩B)

=> P(A) + 2P(A) – 2P(A)P(A) = 0.52

∴ $x + 2x – 2x^2 = 0.52$, x = 0.2 or 1.3. But x can't be 1.3.

∴ x = 0.2

∴ P (B) = 0.4

50. For independent events $A_1, A_2, A_3,..........,A_n$, $P(A_i)$ where i = 1, 2, . . ., n. Then the probability that none of the events will occur is

(a) n/n+1 (b) n-1/n+1 (c) 1/n+1 (d) 1-n/n+1

Ans:

P [nonoccurrence of A_i] = $1-\frac{1}{i+1}=\frac{i}{i+1}$

P [nonoccurrence of any of events] = $\left(\frac{1}{2}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{3}{4}\right)\times......\left(\frac{n}{n+1}\right)=\frac{1}{n+1}$.

51. The number of ways in which a mixed double game can be arranged from amongst 9 married couples if no husband and wife play in the same game is

(a) 756 (b) 1296 (c) 1512 (d) 3024

Ans: We can choose two men out of 9 in $^9C_2$ ways. Since no husband and wife are to play in the same game, two women out of the remaining 7 can be chosen in $^7C_2$ ways. If M_1, M_2, W_1 and W_2 are
chosen, then a team may consist of M1 and W1 or M1 and W2. Thus, the number of ways of arranging the game is

$(^9C_2)(^7C_2)(2) = 36 \times 21 \times 2 = 1512$

52. P and Q are two points 100 km apart. A starts running from P towards Q at 10 km/hr. B starts running from Q at exactly the same time and in the same direction as that of A at 20 km/hr. After an hour, B turns back and changes his speed to 10 km/hr. After another hour, B again turns back and changes his speed to 20 km/hr. He keeps on changing his speed and direction in this manner till the time he meets A. After how much time will A and B meet for the first time?

(a) 30 hours (b) 18 hours (c) 10 hours (d) 20 hours

Ans: (d) A covers 10 km in the first hour while B covers 20 km. As a result the distance between them increases by 10 km. A covers 10 km in the next hour while B covers–10 km. As a result the distance between them decreases by 20 km. In the first two hours the distance between A and B decreases by 10 km.
The time taken by A and B to meet for the first time

=(100/10) x 2= 20 hours

53. The question given below is followed by two statements, A and B. Mark the answer using the following instructions:
Mark (a) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Mark (b) if the question can be answered by using either statement alone.

Mark (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Mark (d) if the question cannot be answered even by using both the statements together.

Q. ABCD is a cyclic quadrilateral in which AB = 8 cm and BC = 15 cm. What is the area of the quadrilateral?

A. AD = CD
B. The length of the diameter of the circumcircle of triangle BCD is 17 cm.

Ans: (d) **From Statement A:**

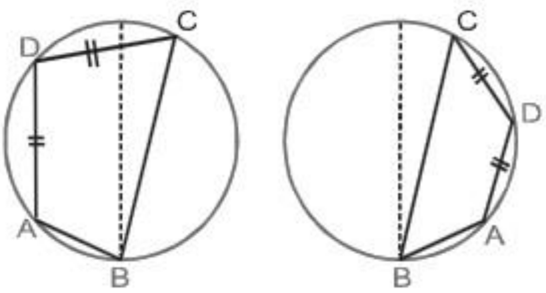
Since we do not know the angle between AB and BC, infinitely many cyclic quadrilaterals ABCD are possible, where AB = 8 cm, BC = 15 cm and AD = CD. Hence, this statement alone cannot answer the question.

From Statement B:

Circumcircle of BCD is also the circumcircle of ABCD. Since we do not know the lengths of AD and CD, infinitely many cyclic quadrilaterals ABCD are possible. Hence, this statement also cannot answer the question alone

Combining Statements A and B:

In a circle of diameter 17 cm, construct a chord BC = 15 cm. This chord divides the circle into two unequal parts. On both these parts, chord AB of length 8 cm can be drawn. Even if AD = CD, we can arrive at two different quadrilaterals ABCD (see the figures given below). Hence, the question cannot be answered even by using both the statements together.



54. A 3-digit natural number 'abc' , where a, b and c are distinct digits, when increased by 33.33% becomes 'cab'. When'cab' is increased by 33.33% it becomes 'bca'. How many such numbers are there?

(a) 0 (b) 1 (c) 2 (d) 5

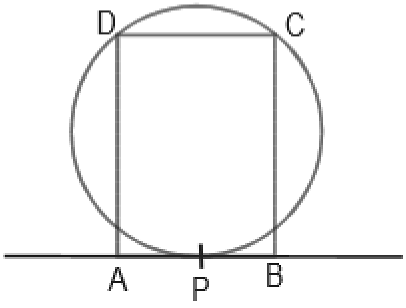
Ans: (c) abc x 1.33 = 4/3 abc = cab ...(i)

cab x 1.33 = 4/3 cab = 16/9 abc = bca ...(ii)

From equation (ii), we can conclude that the resultant number is a multiple of 16 and the initial number is a multiple of 9. Hence, we can say that the resultant number should be a multiple of 16 as well as 9 i.e. a multiple of 144.

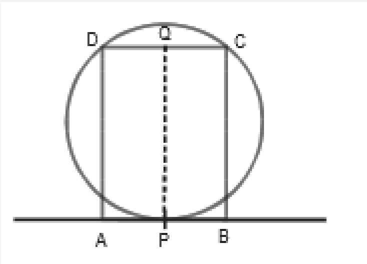
There are two multiples of 144 which satisfy the condition i.e. 432 and 864. Thus the number 'abc' could be either 243 or 486.

55. In the figure given below, a tangent is drawn at point P on a circle of radius 1 cm. A and B are two points on the tangent and ABCD is a rectangle, where C and D are two points on the circumference of the circle. What is the approximate area (in cm^2) of the rectangle ABCD if 2AB = BC?



(a) 1.77 (b) 1.50 (c) 1.83 (d) 1.60

Ans:



AP must be equal to PB.

Let’s assume that the line segment PQ divides the rectangle ABCD into two qual parts (see the figure).

Let AB = 2a; hence, BC = 4a (all lengths in cm).

CP = $\sqrt{(4a)^2 + a^2} = \sqrt{17}a$ = DP

Area of âˆ†CDP = 1/2 PQ.CD = $4a^2$

Radius of the circle = Circumradius of âˆ†CDP

$$= \frac{CD \times CP \times DP}{4(\text{Area of } \Delta CDP)} = \frac{17}{8}a = 1.$$

Hence, a =8/17 cm

Area of rectangle ABCD = 2a× 4a = $8a^2$ = 1.77 cm^2 approximately.

56. In how many ways can 18 identical balls be distributed among 3 identical boxes?

(a) 25 (b) 210 (c) 105 (d) 37

Ans: (i) Let the box with the smallest number of balls does not contain any ball. Then 18 balls can go into 2 identical boxes in 10 ways (0, 18), (1, 17).... (9, 9).

(ii) Let the box with the smallest number of balls contains 1 ball. Then 17 balls can go into 2 identical boxes in 8 ways (1, 16), (2, 15).... (8, 9).

(iii) Let the box with the smallest number of balls contains 2 balls. Then 16 balls can go into 2 identical boxes in 7 ways (2, 14), (3, 13).... (8, 8).

(iv) Let the box with the smallest number of balls contains 3 balls. Then 15 balls can go into 2 identical boxes in 5 ways (3, 12), (4, 11).... (7, 8).

(v) Let the box with the smallest number of balls contains 4 balls. Then 14 balls can go into 2 identical boxes in 4 ways (4, 10), (5, 9).... (7, 7).

(vi) Let the box with the smallest number of balls contains 5 balls. Then 13 balls can go into 2 identical boxes in 2 ways (5, 8) and (6, 7).

(vii) Let the box with the smallest number of balls contains 6 balls. Then 12 balls can go into 2 identical boxes in just 1 way (6, 6).

The number of possible ways = 10 + 8 + 7 + 5 + 4 + 2 + 1 = 37

Alternate Method:
Case I:
All the boxes contain an equal number of balls. There is only one possible case i.e. 6, 6 and 6.

Case II:
Exactly two boxes contain an equal number of balls. There are 9 possible cases i.e. (0, 0, 18), (1, 1, 16), (2, 2, 14), (3, 3, 12), (4, 4, 10), (5, 5, 8), (7, 7, 4), (8, 8,2) and (9, 9, 0).

For each of these cases 3 combinations were possible had the boxes been non-identical.

Case III:
Each box contains a different number of balls.

Let the number of cases be x.
For each of these cases 6 combinations were possible had the boxes been non-identical.

$$\therefore 1 + 9 \times 3 + 6x = {}^{18+3-1}C_{3-1}$$

$$\Rightarrow 28 + 6x = \frac{20 \times 19}{2} = 190$$

$$\Rightarrow x = 27$$

So the required number of ways = 27 + 9 + 1 = 37

57. One hundred ml of alcohol is mixed with y ml of water. Forty ml of this alcohol-water mixture is added to 2y ml of another alcohol-water mixture whose alcohol concentration is 26%. If the percentage of water in the resultant mixture is 2y%, then what is the value of y?

(a) 30 (b) 40 (c) 20 (d) 25

Ans: Volume of water in the 40 ml taken from the first alcohol-water mixture = (Y/(100+y))x40ml

Volume of water in the 2y ml taken from the second alcohol-water mixture = (1 – 0.26) × 2y = 1.48y ml.

Total volume of the two mixtures taken = (40 + 2y) ml.

Hence, $$\frac{\left(\frac{y}{100+y}\right) \times 40 + 1.48y}{40 + 2y} = \frac{2y}{100}$$

Solving the above equation for y, we get y = 25 or –108 (which is rejected)

Note: Instead of solving for y, the value can also be obtained by simply substituting the options in the last equation.

58. If a and b are real numbers such that a^{ab} = b and a ≠ b , then what is the value of a^b,– b?

(a) –1 (b) 0 (c) 1 (d) 2

Ans: It is given that $a^{a^b} = b$

Putting the value of b in left-hand side, we get

$$a^{a^{a^{a^b}}} = b$$

On repeating the same step n times, we get

$$a^{a^{a^{a^b}}} = b$$

When n tends to infinity, we get

$$a^{a^{a^{\cdot^{\cdot^{\cdot^{b}}}}}} = a^b = b$$

Hence $a^b - b = 0$.

59. A function f(x) is defined for all real values of x as f(x)= (x-1)/(x+1). If y_1 = f(x), y_2 = f(f(x)), y_3 = f(f(f(x))) and so on, then what is the value of y_{501}?

(a) -1/x (b) (x+1)/(x-1) (c) 501x– 1 (d)(x-1)/(x+1)

Ans: y1 = (x-1)/(x+1)

y2 = f(y1) = -1/x

y3 = f(y2) = - (x+1)/(x-1)

y4 = f(y3) = x

y5 = f(y4) = (x-1)/(x+1)

It can be concluded that the given function has the cyclicity of 4 or $y_n = y_{n+4k}$, where k is a whole number.

Hence, y_{501} = y1 = (x – 1)/(x + 1)

60. What is the equation of the straight line which passes through the point of intersection of the straight lines 3x + 4y– 11 = 0 and x + y– 3 = 0 and is parallel to the line 2x + 5y = 0?

(a) 5x– 2y– 12 = 0 (b) 2x + 5y– 12 = 0 (c) 2x + 5y– 9 = 0 (d) 5x + 2y– 9 = 0

Ans: Solving the two linear equations 3x + 4y – 11 = 0 and x + y – 3 = 0, we get x = 1 and y = 2.

Hence, the two lines intersect at the point (1, 2). Any line which is parallel to 2x + 5y = 0 should be of the form 2x + 5y – k = 0 …(i)

where k is a real number.

Putting x = 1 and y = 2 in (i), we get k = 12.

Hence, the equation of the straight line will be 2x + 5y – 12 = 0.

61. If a and b are integers such that $\log_2 (a+b)+\log_2(a-b)=3$, then how many different pairs (a, b) are possible?

(a) 0 (b) 1 (c) 2 (d) 3

Ans: $\log_2 (a+b)+\log_2 (a-b) = 3$

=> $\log_2 (a+b)(a-b) = 3$

=> $\log_2(a^2-b^2) = \log_2 2^3$

=> $a^2 - b^2 = 8$

Solving the above equation for integer values of aand b, we get (a, b) ≡ (3, 1) or (3, –1).

Note: 'a – b' must be greater than zero.

62. A cylindrical pipe of length 75 m, through which water flows at the rate of 54 km/hr, can fill 80% of a cuboidal tank of 118800 m3 capacity in 14 hours. What is the radius (in cm) of the cross section of the pipe?

(a) 20 (b) 25 (c) 50 (d) Cannot be determined

Ans: Let the radius of the cross section of the pipe be r.

Speed (v) at which water flows = 54 km/hr = 54000 m/hr

Rate of water flow = (Cross-sectional area of the pipe) x v

$$\therefore \pi r^2 \times 54 \times 10^3 \times 14 = \frac{80}{100} \times 118800$$

$$\Rightarrow r = \frac{2}{10} m = 20 \text{ cm.}$$

63. A large cube is formed by bringing together 729 smaller identical cubes. Each face of the larger cube is painted with red colour. How many smaller cubes are there none of whose faces is painted?

(a) 216 (b) 256 (c) 343 (d) None of these

Ans: There are 8 smaller cubes (on the corners) which have exactly three sides painted. There are 7 × 12 i.e. 84 smaller cubes (on the edges) which have exactly two sides painted.

There are 7 × 7 × 6 i.e. 294 smaller cubes (on the faces) which have exactly one side painted.

The total number of smaller cubes with at least one side painted = 8 + 84 + 294 = 386

So the total number of smaller cubes with none of the sides painted = 729 – 386 = 343.

Alternate Method:

Each edge of the larger cube is made of 9 smaller cubes. It can be observed that there is another cube whose edge is made of 7 smaller cubes which lies inside this larger cube, such that none of the cubes in

it makes to the surface of the larger cube (and didn't get painted as a result).

The total number of smaller cubes in this cube = 7^3 = 343.

63. If A is the sum of the squares of the first n natural numbers (where n < 100), then for how many values of n will A be divisible by 5?

(a) 40 (b) 60 (c) 59 (d) 39

Ans: For n = 2, n= 4 and n = 5 the values that A assumes are $1^2 + 2^2$, $1^2 + 2^2 + 3^2 + 4^2$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2$ respectively. Each of these is divisible by 5.

For n = 1 or 3, A takes values 1^2 and $1^2 + 2^2 + 3^2$ respectively both of which are not divisible by 5.

So in the set of the 1st 5 natural numbers, 3 numbers are divisible by 5.

For n = 6, 7, 8, 9, 10 A behaves in exactly the same manner as for n = 1, 2, 3, 4, 5 respectively. This pattern repeats for the next set of 5 natural numbers and so on.

So for n = 1 to n = 100, A is divisible by 5, in three-fifths of cases. So for 60 values of n A would be divisible by 5.

Since n < 100 and for n = 100, A is divisible by 5, the total number of alues that satisfy the condition would be 59.

Alternate solution :

Sum of the squares of first n natural numbers is n(n+1)(2n+1)/6 = A

Now n can take 5 types of values i.e. 5k, 5k + 1,5k + 2, 5k – 2 and 5k – 1.

Let's put all the values in A : If n = 5k, A will be divisible by 5.

If n = 5k + 1, A = (5k+1)(5k+2)(10k+3)/6

So A is not divisible by 5.

If n = 5k + 2, A = (5k+2)(5k+3)(10k+5)/6

So A is divisible by 5.

If n = 5k - 2, A = (5k-2)(5k-1)(10k - 3)/6

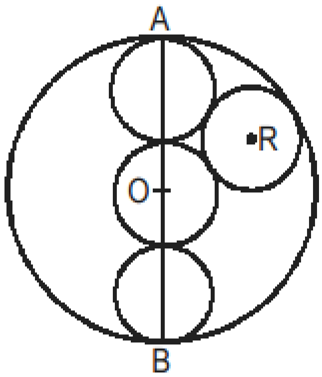
So A is not divisible by 5.

If n = 5k - 1, A = (5k-1)(5k)(10k - 1)/6

So A is divisible by 5

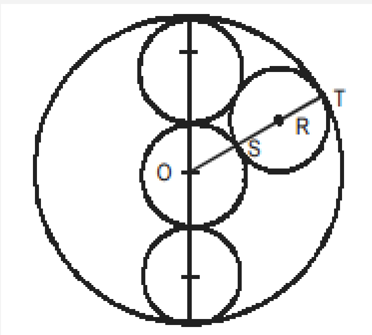
So all the numbers of the type 5k, 5k + 2 and 5k –1 i.e. 3 numbers out of every 5 consecutive numbers will satisfy the given condition. So 57 out of the first 95 natural numbers will satisfy the condition. 97 and 99 also satisfy the given condition. So total numbers are 57 + 2 = 59.

64. In the figure given below, AB is the diameter of the larger circle while three smaller circles are drawn inside this circle such that their diameters are along AB. The radi**63.**us of each of these three circles is 1 cm and the length of AB is 6 cm. Another circle with center at R is drawn which touches the two smaller circles and the larger circle. What is the length of the radius (in cm) of this circle?



a) √3/2 (b)1/√2 (c) 1 (d) None of these

Ans:



Let the radius of the circle with center R be ‘r’ cm.

Note : If two circles touch each other (internally or externally) then the line joining their centers will always pass though the point of contact. The circle with center R and the smaller circle with center O touch each other externally.

Hence, OR = OS + SR = 1 + r ...(i)

Also, OT must pass through R as the circle with center R and the larger circle with center O touch each other internally.

Hence, OT = 3 = OR + RT = 1 + r + r =1 + 2r . ..from (i)

⇒ r = 1 cm.

65. From the first 20 natural numbers how many Arithmetic Progressions of five terms can be formed such that the common difference is a factor of the fifth term?

(a) 16 (b) 22 (c) 25 (d) 26

Ans: Let d be the common difference and a be the first term of AP. The fifth term of the series will be a + 4d. If a + 4d is divisible by d then a should also be divisible by d. Hence the cases are:

d = 1, a = 1, 2, 3,….16

d = 2, a = 2, 4, 6,….12

d = 3, a = 3, 6

d = 4, a = 4

So the answer is 16 + 6 + 2 + 1 = 25.

66. $5f(x)+4f\left(\frac{4x+5}{x-4}\right)=9(2x+1),$ where $x \in R$ and $x \neq 4.$ What is the value of f(7)?

(a) – 17 (b) – 8 (c) – 7 (d) None of these

Ans:

$$5f(x)+4f\left(\frac{4x+5}{x-4}\right)=9(2x+1) \quad ...(i)$$

Putting x = 7 in (i):

5f(7) + 4f(11) = 9 × (2 × 7 + 1)

⇒ 5f(7) + 4f(11) = 135 ...(I)

Put x = 11 in (i):

5f (11) + 4f(7) = 9 x (2 × 11 + 1)

⇒ 5f(11) + 4f(7) = 207 ...(II)

Solving (I) & (II) we get:

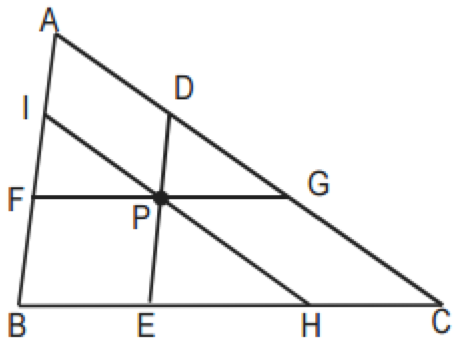
$$f(7) = -\frac{153}{9} = -17$$

67. There were 4 parcels all of whose weights were integers (in kg). The weights of all the possible pairs of parcels were noted down and amongst these the distinct values observed were 94 kg, 97 kg, 101 kg and 104 kg. Which of the following can be the weight of one of the parcels?

(a) 40 kg (b) 45 kg (c) 48 kg (d) 53 kg

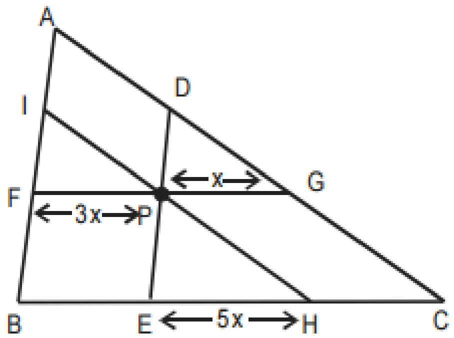
Ans: There are 4 parcels, which would result in ${}^4C_2$ = 6 pairs but it is given that there are only 4 distinct weights. This can only happen when there are some weights which are identical. Out of the 4 numbers here, 2 are odd and 2 are even. So the weight of the identical pair must be either 94 kg or 104 kg. If it is 94 kg, the equal weights must be 47 kg each. This means that the other two weights must be 50 kg and 54 kg. So the 4 parcels will weigh 47 kg, 47 kg, 50 kg and 54 kg. If it is 104 kg, the equal weights must be 52 kg each. This means that the other two weights must be 45 kg and 49 kg. So the 4 parcels will weigh 45 kg, 49 kg, 52 kg and 52 kg.

68. In the figure given below, P is a point inside the triangle ABC. Line segments DE, FG and HI are drawn through P, parallel to the sides AB, BC and CA respectively. The areas of the three triangles DPG, FPI and EPH are 1, 9, and 25 respectively. What is the area of the triangle ABC? (All the areas are in sq cm).



(a) 81 (b) 144 (c) 16 (d) 64

Ans: $FG \parallel BC,\ DE \parallel AB$ and $IH \parallel AC.$ as $FP \parallel BE$ and $BF \parallel EP$, FBEP is a parallelogram.Similarly, ADPI & PGCH are also parallelograms.



âˆ†DPG , âˆ†IFP and âˆ†PEH are similar to âˆ†ABC.

If the area (in sq. cm) of âˆ†DPG , âˆ†IFP and âˆ†PEH are 1, 9 and 25 respectively then we can say their corresponding sides are in the ratio 1 : 3 : 5. Let the lengths (in units) be x, 3x and 5x for the sides PG, FP and EH espectively.

Also BC = BE + EH + HC = FP + EH + PG

BC = 3x + 5x + x = 9x

âˆ†DPG is similar to âˆ† ABC and the ratio of the areas of similar triangles is equal to the ratio of the squares of their corresponding sides,

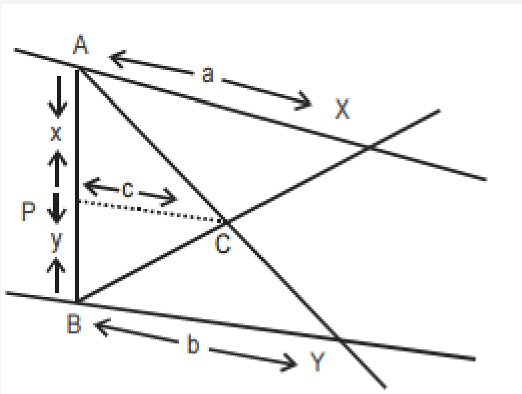
So Area(âˆ†DPG)/Area(âˆ†ABC) $=(X)^2/(9X)^2$
=> 1/Area = $(1/9)^2$ => Area (âˆ†ABC) = 81 sq. cm

69. Guppy has a watch that shows the date without the month and the year. By default, the watch displays 31 days in each month. Therefore, at the end of all the months with less than 31 days the date on the watch needs to be readjusted. On 10th March 2001 it showed the right date as ‘10’. What date would it show on 15th May 2002, if it is known that Guppy never readjusted his watch during this period?
(a) 23 (b) 7 (c) 8 (d) 22

Ans: On 1st April 2001, Guppy’s watch will correctly show the date as ‘1’ as March has 31 days only. From 1st April 2001 to 30th April 2002 a total of 13 months or 365 + 30 = 395 days would have passed.
So the date shown by Guppy’s watch on 30th April, 2002 must be 395 - 12 × 31 = 23. Subsequently his watch will show the date as ‘24’ on 1st May, ‘1’ on 9th May and ‘7’ on 15th May, 2002.

70. Let P be a point on the side AB of a triangle ABC. Lines drawn parallel to PC, through the points A and B, meet BC and AC extended at X and Y respectively. The lengths of AX, BY and PC are
‘a’ units, ‘b’ units and ‘c’ units respectively. Then c will be equal to the half of
(a) Arithmetic Mean of a and b (b) Geometric Mean of a and b (c) Harmonic Mean of a and b (d) None of these

Ans:



Let AP = x, PB = y

Now, âˆ†APC is similar to âˆ†ABY.

Therefore x/(x+y) = c/b ...(i)

Similarly,âˆ†ABX is similar to âˆ†PBC

hence y/x+y = c/a ...(ii)

(i)& (ii)gives

1/c = 1/a + 1/b

c = ab/a+b

c = 1/2(Harmonic mean of a and b)

71. A game consisting of 50 rounds is played among P, Q and R as follows:
Two players play in each round and the player who loses in that round is replaced by the third player in the next round. If the only rounds in which P played against Q are the 3rd, 14th, 25th and 36th, then what can be the maximum number of games won by R?

(a) 40 (b) 42 (c) 41 (d) 36

Ans: If P played against Q in the 3rd, 14th, 25th and 36th rounds, then R must have lost in the 2nd, 13th, 24th and 35th rounds. So games won by R are 50 – 8 = 42, if R has won in the 50th round and games won by
R are 50 – 9 = 41, if R has lost in the 50th round.

72. A is the set of the first 100 natural numbers. What is the minimum number of elements that should be picked from A to ensure that atleast one pair of numbers whose difference is 10 is picked?
(a) 51 (b) 55 (c) 20 (d) 11

Ans: Let's divide the first 100 natural numbers in five sets of 20 numbers each:
{1, 2, 3….20}, {21, 22, 23….40}, ...{81, 82,83……100}. If we pick the first ten numbers from each set we will not get any pair of two numbers whose difference is 10.

However, if we pick just one more number from any of the sets, it would have a difference of 10 with one of the numbers which has already been picked.

So the answer is 10 × 5 + 1 = 51.

73. (X + 3)/3, (X + 8)/4, (X + 15)/5, (X + 24)/6 ...(X + 80)/10 is a sequence where X ≠ 1.

What is the least value of X for which HCF (Numerator, Denominator) = 1 for each term of the given sequence?

(a) 17 (b) 13 (c) 11 (d) None of these

Ans: The general term is of the form (X + n(n + 2))/(n + 2) n(n + 2) is always divisible by (n + 2). So we can say that n(n + 2) 1 ± would never be divisible by (n + 2). If we put X = –1, the numerator and denominator of all the terms would be co-prime.

74. What is the number of non-negative integer solutions for the equation x2 – xy + y2 = x + y?

(a) 3 (b) 4 (c) 1 (d) None of these

Ans: (x2 – xy + y2) = (x + y)

Multiplying both sides by 2:

2(x2 – xy + y2) = 2(x + y)

(x – y)2 + x2 + y2 = 2x + 2y

$$(x-y)^2 + (x-1)^2 + (y-1)^2 = 2$$

$(x-y)^2$ ↓	$(x-1)^2$ ↓	$(y-1)^2$ ↓	
0	1	1	→ (A)
1	0	1	→ (B)
1	1	0	→ (C)

Integer solutions for (x, y):

Case 1: (0, 0) and (2, 2)
Case 2: (1, 2) and (1, 0)
Case 3: (2, 1) and (0, 1)
So there are six non-negative integer solutions.

75. A sequence of non-negative integers is given such that t1 = 150 and n n - 2 n - 1 t = t – t for n > 2. For what value of t2 would the sequence have the maximum possible number of terms?

(a) 90 (b) 97 (c) 93 (d) 75

Ans: All the terms of the sequence have to be non-negative integers. As soon as we get a negative term it would mean that the sequence terminates at the previous term.
Let's write the first few terms:

$t_3 = 150 – t_2$ $t_4 = 2t2 – 150$

t_5 = 300 – 3t2 t_6 = 5t2 – 450

t_7 = 750 – 8t2 t_8 = 13t2 – 1200

t_9 = 1950 – 21t2 t_{10} = 34t2 – 3150

Now let's try to make as many of them positive as possible:

150 – $t_2 \geq 0$ or 150 $\geq t_2$

$2t_2$ – 150 ≥ 0 or $t_2 \geq 75$

300 – 3t2 ≥ 0 or 100 ≥ t_2

$5t_2$ – 450 ≥ 0 or $t_2 \geq 90$

750 – 8t2 ≥ 0 or 93.75 ≥ t_2

$13t_2$ – 1200 ≥ 0 or $t_2 \geq 92.30$

So t_2 must be greater than 92 and less than 94, for the first 8 terms to be positive.

So when t_2 = 93, the sequence would have exactly 8 terms.

For every other value of t2 the number of terms would be less than 8.

So the answer is 93.

76. Anshul and Nitish run between point A and point B which are 6 km apart. Anshul starts at 10 a.m. from A, reaches B, and returns to A. Nitish starts at 10:30 a.m. from B, reaches A, and comes back to B. Their speeds are constant with Nitish's speed being twice that of Anshul's. While returning to their starting points they meet at a point which is exactly midway between A and B. When do they meet for the first time?

(a) 1 10 : 33 1/3a.m. (b) 2 10 : 37 2/3 a.m. (c) 10 : 33 a.m. (d) 2 10 : 33 2/3 a.m

Ans: Let the speed of Anshul be v km/hr. So the speed of Nitish would be 2v km/hr. Time taken by Anshul and Nitish to reach exactly midway between A and B, while returning to their starting points = 9 /V hrs and 9/2V hrs respectively Anshul started 1/2hr early.

Hence 9/V = 9/2V + 1/2 => V=9 km/hr

Distance covered by Anshul till 10:30 a.m.

= 9x1/2 = 4.5 km

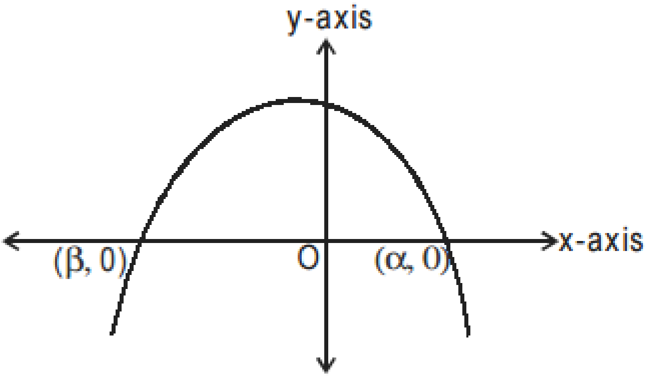
Time required by Anshul and Nitish to cover the remaining 6 – 4.5 = 1.5 km for their first meeting

$$= \frac{1.5}{v+2v} = \frac{1.5}{9+18}\text{hrs} = \frac{1}{18}\text{hrs} = \frac{60}{18}\text{minutes} = 3\frac{1}{3}\text{minutes}$$

Time when they meet first

$$= 10:\left(30+3\frac{1}{3}\right) = 10:33\frac{1}{3}\text{a.m.}$$

77. The graph of $y = ax^2 + bx + c$ is shown below. If it is given that | α |< |β |, then which of the following is true?



(a) a < 0, b < 0, c < 0 (b) a < 0, b > 0, c > 0
(c) a < 0, b < 0, c > 0 (d) a < b, b > 0, c < 0

Ans: As the graph is downward open, so a < 0.
Also, sum of the roots α+β = a/b is negative (or less than zero).

[Since |α |< |β | and β is less than zero α<β] b/a< 0

So b < 0.

Also, product of the roots is negative as β is negative.
So c
a αβ = is negative or c 0. a <
So c is positive (or greater than zero).
Hence a < 0, b < 0 and c > 0.

78. A and B are moving along the circumference of a circle with speeds that are in the ratio 1 : K. They start simultaneously from a point P in the clockwise direction. They meet for the first time at a point Q which is at a distance of one-third the circumference from P, in the clockwise direction. K cannot be equal to
(a)1/4 (b)4/7 (c) 4 (d) None of these

Ans: Since A and B are moving in the same direction the faster one takes a lead of one circle over the slower one when they meet for the first time after the start.

If A is faster than B:

1) They meet for the first time when A finishes 4/3 rounds and B finishes 1/3 rounds.

In this case K = 1/4 and the ratio 1 : K = 4 : 1

2) They meet for the first time when A finishes 7/3 rounds and B finishes 4/3 rounds.

In this case K = 4/7 and the ratio 1 : K = 7 : 4

If B is faster than A:

They meet for the first time when A finishes 1/3 rounds and B finishes 4/3 rounds.

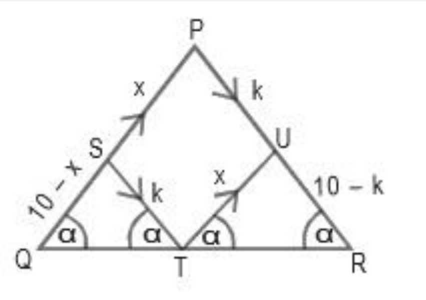
In this case K = 4 and the ratio 1 : K = 1: 4

So all the given values of K are possible.

79. In triangle PQR, PQ = PR = 10 cm. Points S, T and U lie on PQ, QR and PR respectively such that ST is parallel to PR and UT is parallel to PQ. What is the perimeter (in cm) of the quadrilateral PSTU?

(a) 18 (b) 20 (c) 24 (d) Data Insufficient

Ans:



Let PS = x cm and PU = k cm.

Now ST||PR

=>

Also, since PQ = PR

∠PQR =∠PRQ = α egree (isoceles triangle) Therefore in âˆ†SQT :

Since PSTU is a parallelogram, PU = ST

Perimeter (PSTU) = 2k + 2x = 2(10 – x) + 2x = 20 cm.

Alternate Solution:
Take S and T at Q. Perimeter will be 10 + 0 + 10 + 0= 20 cm.

80. If 'x' is a real number then what is the number of solutions for the equation $\sqrt{(x^4 + 16)} = x^2 - 4$?

(a) 0 (b) 1 (c) 2 (d) 3

Ans: $x^4 + 16$ is always greater than x4 and x^2 is always greater than $x^2 - 4$. Hence, $\sqrt{x^4}$ will always be greater than $x^2 - 4$. So$\sqrt{(x^4 + 16)}$ is greater than x2 – 4.
So the given two expressions can never be equal for any real value of x.

81. N! is completely divisible by 13^{52}. What is sum of the digits of the smallest such number N?

(a) 11 (b) 15 (c) 16 (d) 19

Ans: The number needs to be less than 13 × 52 = 676. The highest power of 13 in 676! is 56.
The power of 13 in the smallest such number needs to be exactly 52. If we subtract 13 × 3 = 39 from 676, we get 637. The number 637! will be the smallest number of type N! that is completely divisible by 1352.

The sum of the digits of 637 is 16.

82. The lengths of the three edges of a cuboid are increased by a%, b% and c%. The volume increases by V%, where V is an integer. How many values can V take if a, b, c are real numbers and 10 ≤ a, b, c ≤ 20?

(a) 11 (b) 39 (c) 41 (d) Cannot be determined

Ans: The increase in volume will be minimum when a, b and c are 10% each.
The new volume will be 1.1 × 1.1 × 1.1 = 1.331 times of the old volume. So the percentage increase in volume will be 33.1%.

Similarly, the increase in volume will be maximum when a, b and c are 20% each.

The new volume will be 1.2 × 1.2 × 1.2 = 1.728 times of the old volume. So the percentage increase in volume will be 72.8%.

As the final percentage increase in volume is an integer, the value must be an integer from 34 to 72 i.e. 39 distinct values are possible.

83. The question given below is followed by two statements, A and B. Mark the answer using the following instructions:
Mark (a) if the question can be answered by using either statement alone.
Mark (b) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.
Mark (c) if the question cannot be answered even by using both the statements together.
Mark (d) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Q. In a class of 200 students, the highest and the lowest scores in a test are 98 and 18 respectively. Is 50 the average score of the class in the test?
A. 100 students score above 50 and the remaining 100 students score below 50 in the test.
B. If the highest score and the lowest score in the test are excluded, the sum of the top 99 scores is exactly double of the sum of the bottom 99 scores.
Ans: **From Statement A:**
The average score of the class cannot be calculated as neither the total scores nor the average scores of the two groups are known.

From Statement B:
The data is insufficient to calculate the exact average score of the class.

From Statements A and B together:
Combining the two statements also does not result in anything conclusive about the average score of the class.

84. What is the total number of ways of selecting twenty balls from an infinite number of blue, green and yellow balls?
(a) 3^{20} (b) 20^3 (c) 231 (d) 1771

Ans: Let the number of blue, green and yellow balls picked be x, y and z respectively.
$\therefore x + y + z = 20$

So the number of ways = ${}^{20+3-1}C_{3-1} = {}^{22}C_2 = 231$

85. In a class comprising 60 boys and some girls, the average age of boys is 14.8 years and that of girls is 14.1 years. If the average age of the class is 14.7 years, then how many girls are there in the class?
(a) 10 (b) 15 (c) 20 (d) 25

Ans: Let the number of girls in the class be n.

∴ $(60\times14.8 + 14.1\times n)/(60 + n) = 14.7$

$n = 10$

86. If m and n are positive integers such that$(m – n)^2 = 4mn/(m + n – 1)$, then how many pairs (m, n) are possible?

(a) 4 (b) 10 (c) 16 (d) Infinite

Ans: $(m–n)^2 = 4mn/(m+n-1)$

$\Rightarrow (m–n)^2 (m+n-1) = 4mn$

$\Rightarrow (m–n)^2 (m+n-1) = (m+n)^2 - (m–n)^2$

$\Rightarrow (m–n)^2 (m+n-1) = (m+n)^2$

$\Rightarrow (m–n)^{2\,=}(m+n)$

(Since, $m + n \neq 0$)

The above equation has infinitely many solutions where m and n are positive integers.

We can put m + n = v and m – n = u, and re-write the equation as u2 = v and then plug in different values of u and v to get different pairs of (m, n).

87. The lengths of the hour hand and the minute hand of a clock are 3.5 cm and 5.25 cm respectively. If the hour hand covers an area of 7.7 cm2, then find the approximate area (in cm^2) covered by the minute hand during the same time period.

(a) 17 (b) 158 (c) 260 (d) 208

Ans: Let θ be the angle made by the hour hand when the area covered by it is 7.7 cm^2

$\Rightarrow \theta = 7.7x360^0/\pi x(3.5)^2 = 72^0$

As the speed of minute hand is 12 times the speed of hour hand, the angle covered by the minute hand in the same time will be 12 × θ i.e. 864^0 Area covered by minute hand

$= 864^0/360^0 \times 22/7 \times (5.25)^2 = 208\ cm^2$

88. In âˆ†ABC, M is the midpoint of AB and N is the midpoint of AC. CM and BN meet at point O and are perpendicular to each other. The length of AB is $2\sqrt{13}$cm and that of AC is $\sqrt{73}$ cm. What is the length of BC (in cm)?

(a) 17 (b) 19.25 (c) 8 (d) 5

Ans: Let the lengths (in cm) of NC and MB be ‘b’ and ‘c’ respectively

∴$b = \sqrt{73}/2$ and $c = \sqrt{13}$

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long as the third side.

∴âˆ†MON - âˆ†COB

=> MO/CO = ON/OB = MN/BC = 1/2

In right angled triangles MOB and NOC, by Pythagoras theorem

$c_2 = MO^2 + BO^2 = MO^2 + 4ON^2$
$B^2 = OC^2 + NO^2 = 4MO^2 + ON^2$
$=> MO^2 + ON^2 = (b^2 + c^2) / 5$
In right angled triangle MON, by Pythagoras theorem

$MN = \sqrt{MO_2} + ON^2 = \sqrt{(b^2 + c^2)/5} = \sqrt{25/4} = 5/2$ cm
⇒ BC = 2MN =5 cm
89. There are 13 equidistant bus stops on a straight road. A bus running at 60 km/hr is some distance away from the 1st stop from where it will move towards the 13th stop. Two cars start running from the 6th stop in opposite directions with the same speed. If the bus meets one of the cars at the 1st stop and the other at the 13th stop, then find the speed of the cars.
(a) 10 km/hr (b) 20 km/hr (c) 30 km/hr (d) Cannot be determined

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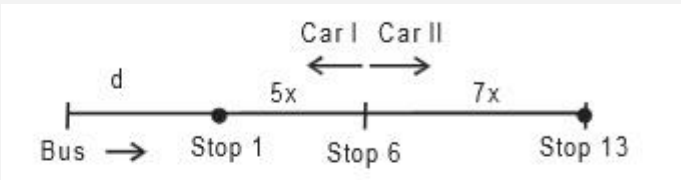
Ans:



Let the speeds of the two cars be s km/hr and the distance travelled by them be 5x and 7x km respectively. Let the distance travelled by the bus be d km.

∴ d/60 =5x/s ⇒ d/(60x5) = ×/5 ..(i)

and (d+12x)/60 = 7x/s => (d+12x)/(60x7) = x/s ...(ii)

From (i) and (ii), we get d = 30x

∴ 30x/60 = 5x/s ⇒ s = 10

90. How many divisors of 25200 can be expressed in the form 4n + 3, where n is a whole number?

(a) 6 (b) 8 (c) 9 (d) None of these

Ans: $25200 = 2^4 \times 3^2 \times 5^2 \times 7^1$

As the required divisors when divided by 4 leave remainder 3, the power of 2 in the divisors has to be 0.Therefore, any such divisor is of the form $3^a \times 5^b \times 7^c$,which when divided by 4 leaves the remainder$(-1)^a \times 1^b \times (-1)^c$.

For the remainder to be 3 i.e.–1, one of 'a' or 'c' must be even/0 and the other should be odd. Also, 'b' can take all the three possible values without making a difference to the remainder. The nine possibilities are listed below:

a = 0, b = 0, c = 1
a = 2, b = 0, c = 1
a = 1, b = 0, c = 0
a = 0, b = 1, c = 1
a = 2, b = 1, c = 1
a = 1, b = 1, c = 0
a = 0, b = 2, c = 1
a = 2, b = 2, c = 1
a = 1, b = 2, c = 0

91. The HCF of three natural numbers x, y and z is 13. If the sum of x, y and z is 117, then how many ordered triplets (x, y, z) exist?

(a) 28 (b) 27 (c) 54 (d) 55

Ans: Let the three numbers be 13a, 13b and 13c, where a, b and c are coprime.

∴ 13a + 13b + 13c = 117

⇒ 13(a + b + c) = 13 × 9

⇒ a + b + c = 9

The number of positive integer solutions of a + b + c = 9 is ${}^{9-1}C_{3-1}$ i.e.${}^{8}C_{2}$ = 28.

However, there is a case, a = b = c = 3, where a, b and c are not coprime.

So the answer = 28 – 1 = 27

92. n is a natural number such that ${}^{n}C_{4}={}^{n}C_{12}$. What is the remainder when n! is divided by n + 1?

(a) n – 1 (b) n – 2 (c) n (n) 0

Ans: As ${}^{n}C_{4} = {}^{n}C_{12}$, n = 16. So we need to find the remainder when 16! is divided by 17.

(p– 1)! + 1 is divisible by p if p is a prime number.

Hence, 16! will leave remainder –1 i.e. 16 when divided by 17.

93. What is the number of common tangents of the circles $x^2 + y^2 – 2x – 2y – 23 =$ and $x^2 + y^2 – 12x – 26y + 141 = 0$?

(a) 0 (b) 2 (c) 3 (d) 4

Ans: Let $C_1 = x^2+y^2-2x-2y-23 = 0$ and $c_2 = x^2+y^2-12x-26y+141 = 0$

Let points O_1 and O_2 be the centers and r_1 and r_2 be the radii of the circles C_1 and C_2 respectively

Hence, $O_1 = (1,1)$ and $O_2 = (6,13)$.

$O_1O_2 = \sqrt{(6-1)^2 + (13-1)^2} = 13$ units

$r_1 = \sqrt{(1)^2 + (1)^2 -(-23)} = 5$ units

$r_2 = \sqrt{(6)^2 + (13)^2 -(141)} = 8$ units

$r_1 + r_2 = 5 + 8 = 13 = O_1O_2$

The distance between the centers of the two circles is the same as the sum of the radii of the two. So the two circles touch each other externally and the number of common tangents will be 3

94. $U = 5(\log_2 x)^2 – 5(\log_2 x) – 8$, where x is a real number. If $x^U = 16$, find the value of x.

(a) 1 (b) 2 (c) 4 (d) 8

Ans: It is given that $x^U = 16$.

Taking log to the base 2 on both the sides, we get

$U \log_2 x = \log_2 16 = 4$

$\Rightarrow U = 4/\log_2 x$

Let us assume the value of $\log_2 x$ to be y, therefore, $U = 4/y$.

Now putting this value in equation

$U = 5(\log_2 x)^2 - 5(\log_2 x) - 8$, we get

$4/y = 5y^2 - 5y - 8$

$\Rightarrow 5y^3 – 5y^2 – 8y – 4 = 0$

$\Rightarrow 5y^2(y – 2) + 5y (y – 2) + 2 (y – 2) = 0$

$\Rightarrow (y – 2) (5y^2 + 5y + 2) = 0$

$\therefore$ y = 2, as other roots are not real.

$\Rightarrow \log_2 x = 2$

$\Rightarrow x = 2^2 = 4$

Note: We can also substitute the options to arrive at the answer.

95. The digits of a 3-digit number in Base 4 get reversed when it is converted into Base 3. How many such numbers exist?

(a) 0 (b) 1 (c) 2 (d) 3

Ans: Let the 3-digit number be abc. Now according to the given condition, $(abc)_4 = (cba)_3$.

16a + 4b + c = 9c + 3b + a

⇒15a + b = 8c

The only set of numbers which satisfies the relation

given above is a = 1, b = 1 and c = 2

96. A = {3, 23, 43 ....603} and S is a subset of A. If the sum of no two elements of S is more than 606, then what can be the maximum possible number of elements in S?

(a) 15 (b) 14 (c) 17 (d) 16

Ans: The terms of set A form an A.P. with first term 3 and common difference 20.

The number of terms in set A = ((603-3)/20) + 1 = 31

Let a^{th} and b^{th} terms of set A be the largest and the second largest terms of set S.

∴ 3 + 20(a-1) + 3 + 20(b-1) ≤ 606

=> 20(a+b) + 6 - 40 ≤606

=> a + b ≤ 32

∴ Maximum (a, b) = (16, 15)

Thus, sum of any two elements of set A up to the 16^{th} term will always be less than 606.

Hence, the maximum possible number of elements in set S is 16.

97. The solution set for |5x + 2| ≤ 10 is

(a) 8/5≤ x ≤12/5 (b)-12/5≤ x ≤-8/5 (c) -8/5≤ x ≤12/5 (d) -12/5≤ x ≤ 8/5

Ans:

$$|5x+2| = \pm(5x+2)$$
$$\Rightarrow 5x+2 \le 10 \text{ and} -(5x+2) \le 10$$
$$\Rightarrow -10 \le 5x+2 \le 10$$
$$\Rightarrow \frac{-12}{5} \le x \le \frac{8}{5}$$

98. There are two Arithmetic Progressions A and B such that their n^{th} terms are given by A_n= 101 + 3(n– 1) and B_n = 150 + (n – 1), where n is the set of natural numbers. The first 50 terms of A and B are written alternately i.e. $A_1B_1A_2B_2.....A_{50}B_{50}$. What is the remainder when the number so formed is divided by 11?

(a) 0 (b) 1 (c) 9 (d) 10

Ans: The number so formed is 101150...248199. We can write this number as:

$101 \times 10^{297} + 150 \times 10^{294} + . . . + 248 \times 10^3 + 199 \times 10^0$

When 10^n is divided by 11, the remainder is 1 if n is even and the remainder is –1 if n is odd.

Thus, the remainder when the number is divided by 11

= –101 + 150 –104 +151...– 248 +199

= – (101 + 104 + 107...+ 248) + (150 + 151...+ 199)

= - (101 + 248/2)x50 + (150 + 199/2) x 50

= - (349/2) x 50 + (349/2) x 50 = 0

99. How many 4-digit multiples of 3 can be formed using the digits 2 and 3 only?

(a) 4 (b) 6 (c) 5 (d) 7

Ans: The sum of the digits of the 4-digit numbers could be:

i. 2 + 2 + 2 + 2 = 8

ii. 2 + 2 + 2 + 3 = 9

iii. 2 + 2 + 3 + 3 = 10

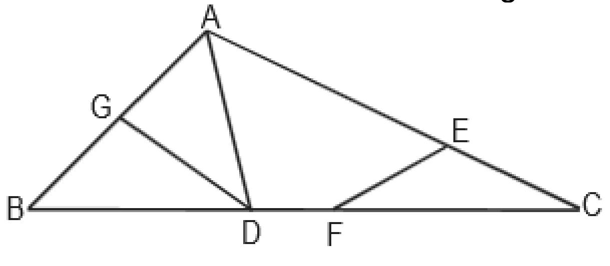
iv. 2 + 3 + 3 + 3 = 11

v. 3 + 3 + 3 + 3 = 12

Only cases (ii) and (v) can be taken for the numbers to be divisible by 3.

Total such numbers = 4!/3! + 1 = 5.

100. In the figure given below, BG = GA = GD, AD = BD and EF = EC. Also, ADFE is a cyclic quadrilat-eral. Which of the following statements is/are definitely true?



(i) The orthocentre of triangle ABC lies at point A.

(ii)âˆ†GBD and âˆ†GDA are congruent.

(iii)AD is a median of triangle ABC

(iv) AD/EF = √2

(a) (i)and(iii)   (b)(i),(ii)and(iii)   (c)(ii),(iii)and(iv)   (d)All four are true

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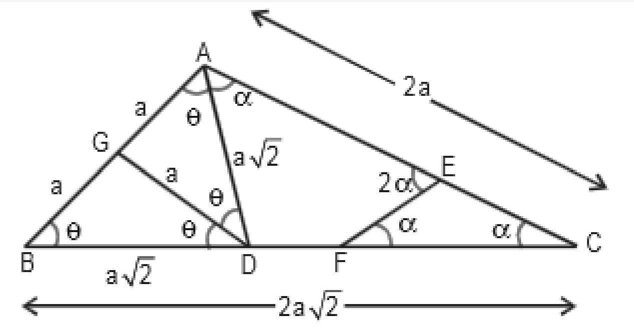
Ans:



Let AG = BG = GD = ‘a’ units

In triangle ABD,θ+θ+2θ= 180°⇒θ= 45°

As exterior angle of a cyclic quadrilateral is equal to the interior opposite angle, angle DAE = angle EFC and angle AEF = angle ADB.

Hence,α = θ = 45°

(i) âˆ†ABC is a right angled isoceles triangle and so its orthocentre lies at A.

(ii) âˆ†GBD ≅ âˆ†GDA

(iii) Since AD = BD and AD = DC, BD = DC. Thus AD is a median of âˆ†ABC.

(iv) In right angled isoceles triangle EFC, let EF = EC 'be’ units; therefore, FC = b√2 units.

=> DF = a√2 - b√2 , which must be greater than 0.

Hence,a√2/b > √2 or AD/EF > √2